1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x$

$y'' - y' - 2y = 0$

$y = e^{kx} \quad y' = ke^{kx} \quad y'' = k^2e^{kx}$

$k^2e^{kx} - ke^{kx} - 2e^{kx}$

$k^2 - k - 2 = 0$

$(k^2 + k) - (2k - 2) = 0$

$k(k+1) - 2(k+1) = 0$

$(k+1)(k-2) = 0$

$k = -1 \quad k = 2$

$y = Ae^{k_1x} + Be^{k_2x} \therefore y = Ae^{-x} + Be^{2x}$

$dy/dx = 0 \quad d^2y/dx^2 = 0$

$0 - 0 - 2c = x$

$-2c = x$

$c = x/-2$

$c = -4$

$= Ae^{-x} + Be - 4$

2. $d^2y/dx^2 - 4y = 10e^{3x}$

$y'' - 4y = 0$

$y = e^{kx} \quad y'' = k^2e^{kx}$

$k^2e^{kx} - 4e^{kx} = 0$

$k^2 - 4 = 0$

$k = \sqrt{4}$

$m = \pm 2$

$y = A\cosh 2x + B\sinh 2x$

~~$dy/dx = A$~~ $y = Ae^{3x}$

$y' = 3Ae^{3x}$

$y'' = 9Ae^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$5Ae^{3x} = 10e^{3x}$

$A = 10e^{3x} / 5e^{3x}$

$A = 2$

$y = 2e^{3x}$

$\therefore A\cosh 2x + B\sinh 2x + 2e^{3x}$.

$4 - D = -3$

$-D = -7$

$D = 7$

$y = -2x + 7$

$y = Ae^{-3x} + Be^{7x} - 2x + 7$

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 0$

$k^2 - 2k - 4k + 8 = 0$

$k(k-2) - 4(k-2) = 0$

$k_1 = 2, \quad k_2 = 4$

$y = Ae^{2x} + Be^{4x}$

$y = ce^{4x}$

$\frac{dy}{dx} = 4ce^{4x}$

$y'' = 16ce^{4x}$

$16ce^{4x} - 6(4ce^{4x}) + 8(ce^{4x}) = 8e^{4x}$

$ce^{4x}(16 - 24 + 8) = 8e^{4x}$

$ce^{4x}(0) = 8e^{4x}$

$c = \frac{8e^{4x}}{e^{4x}(0)}$

$y = 0$

$4Ae^{-2x} + 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-x}$

$Ce^{-2x} = 2e^{-x}$

$C = 2$

$y = 2e^{-x}$

$y = [e^{-2x}(A\cos x + B\sin x)] + 2e^{-(x)}$

$1 = 1(A+0)+2$

$1 = A+2$

$A = -1$

$x = 0 \quad dy/dx = -2$

$y' = -2e^{-2x}(A\sin x + B\cos x) + 2e^{-x}$

$-2 = -2e^{-2(0)}(-A\sin(0) + B\cos(0)) - 2e^{-x}$

$-2 = -2(+0) - 2$

$-2 = -2B - 2$

$B = 0$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-x}$

$y = -e^{-2x}\cos x + 2e^{-x}$

7 $3d^2y/dx^2 - 2dy/dx - y = 2x - 3$

$3d^2y/dx^2 - 2dy/dx - y = 0$

$3y^2 - 2y' - y = 0$

$3y^2 - 3y + y - 1 = 0$

$3y(y-1) + 1(y-1) = 0$

$y = 1, \quad y = -1/3$

$y = Ae^{x} + Be^{-1/3}$

$y = Cx + D$

$dy/dx = C$

$d^2y/dx^2 = 0$

$3(0) + (-2(C)) - (Cx + D) = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

$-2C - D - Cx = 2x - 3$ Comparing Coeff

$-2(-2) - D = -3$

$-C = 2$

$\therefore C = -2$

$-2(-2) - D = -3$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$[-C\cos x - D\sin x] - 2[-C\sin x + D\cos x] + C\cos x + D\sin x = 4\sin x$

$-C\cos x + 2C\sin x + C\cos x - 2D\sin x + 2D\cos x = 4\sin x$

$\cos x[-C - 2D + C] + \sin x[2C + 2D] = 4\sin x$

$\cos x[-2D] + \sin x[2C + 2D] = 4\sin x$

Comparing coefficients:

$-2D = 0 \quad (1)$

$2C + 2D = 4 \quad (2)$

$C = 4/2$

$C = 2$

$-2D = 0$

$D = 0$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

$y = [Ae^x + Be^x] + 2\cos x$

5. $d^2y/dx^2 + 4dy/dx + 5y = 2e^{-2x}$

given that @ $x = 0$, $y = 1$ & $dy/dx = -3$

$y'' + 4y' + 5y = 0$

$k^2 + 4k + 5 = 0$

$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$k = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$

$k = \frac{-4 \pm \sqrt{-4}}{2}$

$k = -4/2 \pm j\,2/2$

$k = -2 \pm j$

$y = Ae^{-2x}\cos x + Be^{-x}\sin x$

$y = Ae^{-x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

$4Ae^{-2x} + (-2Ae^{-x}) + 5(Ae^{-2x}) = 2e^{-2x}$

3 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$y'' + 2y' + y = 0$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

$k^2 + 2k + 1$

$k^2 + k + k + 1 = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$y = Ae^{kx} + Bxe^{kx}$

$y = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$P.I = e^{-2x}$

$\therefore e^{-2x}(Ae^{-2x} + Bxe^{-2x})$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$y'' + 25y = 0$

$k^2 + 0 + 25 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{0 \pm \sqrt{0^2 - 4(1)(25)}}{2(1)}$

$= \frac{0 \pm \sqrt{-25 \times 4}}{2}$

$= \frac{0 \pm \sqrt{1} \cdot \sqrt{4} \cdot \sqrt{25}}{2}$

$k = \frac{\pm j 2\sqrt{25}}{2}$

$k = \pm j\sqrt{25}$

$k = \pm j5$

$y = A\cos 5x + B\sin 5x$

$P.I: y = Ax^2 + Bx + C$

$\frac{dy}{dx} = 2Ax + B$

$d^2y/dx^2 = 2A$

$2A + B(2Ax + B) + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25(x^2 + 3x + C) = 5x^2 + x$

$2A + 25x^2 + 75Bx + 25C = 5x^2 + x$

$2A + 25C = 0$ —(1)

$25B = 5$ —(2)

$25C = 1$ —(3)

From eqn 1

$2A + 25 = 0$

from 2

$25B = 5$

$A = 1/5$

from 3

$25C = 1$

$B = 1/25$

sub $1/5$ into eqn 1

$2A + 25C = 0$

$2(1/5) + 25C = 0$

$25C = -2/5$

$C = -2/5 + 1/25$

$C = -2/125$

$\therefore y = 1/5x^2 + 1/25x - 2/125$

$y = A\cos 5x + B\sin 5x + 1/5x^2 + 1/25x - 2/125$

5. $d^2y/dx^2 - 2dy/dx + y = 4\sin x$

$d^2y/dx^2 - 2dy/dx + y = 0$ $\quad y = e^{kx}$ $\quad y' = ke^{kx}$ $\quad y'' = k^2e^{kx}$

$y'' - 2y' + y = 0$

$k^2 - k - k + 1 = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$ twice

$k = 1$

$y = Ae^{1x} + Bxe^{1x}$

$y = (\cos x + A\sin x$

$y' = -(\sin x + 17\cos x$