EJIM, CHIMGOZIRIM PRINCE
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PETROLEUM ENGINEERING

Assignment 1

01. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Assuming homogenity,

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Let $y = e^{kx}$; $dy/dx = ke^{kx}$; $d^2y/dx^2 = k^2e^{kx}$

$\therefore k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$

$e^{kx}(k^2 - k - 2) = 0$

$\therefore k^2 - k - 2 = 0$

Factorising by completing the squares;

$k^2 - k = 2$

$k^2 - k + (-\frac{1}{2})^2 = 2 + \frac{1}{4}$

$(k - \frac{1}{2})^2 = \frac{9}{4}$

$k = \pm\sqrt{\frac{9}{4}} + \frac{1}{2}$

$\therefore k_1 = \frac{3}{2} + \frac{1}{2} = 2$; $k_2 = -\frac{3}{2} + \frac{1}{2} = -1$

but $y = C_1e^{k_1x} + C_2e^{k_2x}$

$\therefore y_{homogenous} = C_1e^{2x} + C_2e^{-x}$

Let $y_{non-homogenous} = A$

$\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 0$

$\therefore 0 - 0 - 2A = 8$

$\therefore A = -4$

Hence, $y_{non} = -4$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1e^{2x} + C_2e^{-x} - 4$.

02. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Assuming homogenity,

$\frac{d^2y}{dx^2} - 4y = 0$

Let $y = e^{kx}$, $\frac{d^2y}{dx^2} = k^2e^{kx}$

$k^2e^{kx} - 4e^{kx} = 0$

$\therefore k^2 = 4$; $k = \pm 2$

$\therefore k_1 = 2$, $k_2 = -2$

Hence, $y_{homo} = C_1\cosh 2x + C_2\sinh 2x$

Let $y_{non} = Ae^{3x}$

$\frac{dy}{dx} = 3Ae^{3x}$, $\frac{d^2y}{dx^2} = 9Ae^{3x}$

∴ ~~$y \to 9Ae^{3x} - 12Ae^{3x}$~~

∴ $9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$5Ae^{3x} = 10e^{3x}$

$5A = 10$

∴ $A = 2$

Hence, $y_{non} = 2e^{3x}$

but $y = y_{homo} + y_{non}$

$= C_1\cosh 2x + C_2\sinh 2x + 2e^{3x}$.

03 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = 0$

∴ $k^2 + 2k + 1 = 0$

By completing the squares;

$k^2 + 2k = -1$

$k^2 + 2k + 1^2 = -1 + 1$

$(k+1)^2 = 0$

∴ $k = -1$

but $y = c_1e^{kx} + xc_2e^{kx}$

∴ $y_{homo} = C_1e^{-x} + xC_2e^{-x} = e^{-x}(C_1 + xC_2)$

Let $y_{non} = Ae^{-2x}$

$y' = -2Ae^{-2x}$, $y'' = 4Ae^{-2x}$

∴ $4Ae^{-2x} - 4Ae^{-2x} = e^{-2x} - Ae^{-2x}$

∴ $A = 1$

Hence, $y_{non} = e^{-2x}$

but $y = y_{homo} + y_{non}$

∴ $y = e^{-x}(C_1 + xC_2) + e^{-2x}$.

Q4 $\dfrac{d^2y}{dx^2} + 25y = 5x^2 + x$

Assuming $y = e^{kx}$, $y'' = k^2 e^{kx}$

$k^2 + 25 = 0$

$\therefore k = \pm 5i$

$k_1 = 5i$, $k_2 = -5i$

$\therefore y_{homo} = C_1 \cos 5x + C_2 \sin 5x$

Let $y_{non} = Ax^2 + Bx + C$

$y' = 2Ax + B$, $y'' = 2A$

$\therefore 2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$2A + 25C = 0$ —— (1)

$25B = 1$ $\therefore B = \frac{1}{25}$ —— (2)

$25A = 5$ $\therefore A = \frac{1}{5}$ —— (3)

Putting eq. (3) in (1)

$2\left(\frac{1}{5}\right) + 25C = 0$

$\therefore 25C = -\frac{2}{5}$

Hence, $C = -\frac{2}{125}$

$\therefore y_{non} = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$.

Q5 $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 4\sin x$

Let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$k^2 e^{kx} - 2ke^{kx} + e^{kx} = 0$ (Assuming homogenity)

$k^2 - 2k + 1 = 0$

By completing the squares,

$k^2 - 2k = -1$

$k^2 - 2k + (-1)^2 = -1 + 1$

$(k-1)^2 = 0$

$\therefore k = 1$

Hence, $y_{homo} = C_1 e^x + xC_2 e^x$

Let $y_{non} = A\sin x + B\cos x$, $y' = A\cos x - B\sin x$, $y'' = -A\sin x - B\cos x$

$\therefore \cancel{y} \; \cancel{-A\sin x} - \cancel{B\cos x} - 2A\cos x + 2B\sin x + \cancel{A\sin x} + \cancel{B\cos x} = 4\sin x$

~~collecting like terms~~, Collecting like terms: $-2A = 0$, $A = 0$

$2B = 4$, $B = 2$

$\therefore\ y_{non} = 2\cos x$

but $y = y_{homo} + y_{non}$

Hence, $y = C_1 e^{x} + xC_2 e^{x} + 2\cos x$

06 $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 5y = 2e^{-2x}$ {given that at $x=0$, $y' = 1$ and $y'' = -2$}

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$ and $y'' = k^2 e^{kx}$

$\therefore\ k^2 + 4k + 5 = 0$

By completing the squares,

$k^2 + 4k = -5$

$k^2 + 4k + (2)^2 = -5 + 4$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$\therefore\ k = \pm i \mp 2$; $k_1 = i \mp 2$, $k_2 = -i \mp 2$

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$= C_1 e^{(i\mp 2)x} + C_2 e^{(i\mp 2)x}$

$= C_1 e^{ix} e^{2x} + C_2 e^{-ix} \cdot e^{2x}$

$= e^{2x}(C_1 e^{ix} + C_2 e^{-ix})$

$y_{homo} = e^{2x}(C_1 \cos x + C_2 \sin x)$

~~Let $y_{non} = Ae^{-2x}$ Axe^{2x}~~

$y' = -2Ae^{-2x}$ ~~$y'' = 4Ae^{-2x}$~~

$\therefore$ ~~$4Ae^{-2x}$~~ $- 8Ae$

Let $y_{non} = Ae^{-2x}$

$y' = -2Ae^{-2x}$, $y'' = 4Ae^{-2x}$

$\therefore\ \cancel{y}\ 4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$\therefore\ A = 2$

Hence $y_{non} = 2e^{-2x}$

but $y = y_{homo} + y_{non}$

$y = e^{2x}(C_1 \cos x + C_2 \sin x) + 2e^{-2x}$

but at $x = 0$, ~~$y' = 1, y'' = -2$~~ $y = 1$, $y' = -2$

$y = C_1 \cos x\, e^{2x} + C_2 \sin x\, e^{2x} + 2e^{-2x}$

$y' = C_1[\cos x \cdot 2e^{2x} + e^{2x} \cdot -\sin x] + C_2[\sin x \cdot 2e^{2x} + e^{2x} \cdot \cos x] - 4e^{-2x}$

$\therefore\ y' = C_1(2e^{2x}\cos x - \sin x\, e^{2x}) + C_2(2e^{2x}\sin x + e^{2x}\cos x) - 4e^{-2x}$

$y'' = C_1[3e^{2x}\cos x] + C_2[4e^{2x}\cos x + 3e^{2x}\sin x] + 8e^{-2x}$

Inputting the conditions at $x=0$

$y = C_1 \cos(0) e^{2(0)} + C_2 \sin(0) e^{2(0)} + 2e^{-2(0)} = 1$

$C_1 + 2 = 1$

$\therefore C_1 = -1$

$y' = C_1 [2e^{2(0)} \cos(0) - \sin(0) e^{2(0)}] + C_2 (2e^{2(0)} \sin(0) + e^{2(0)} \cos(0)] - 4e^{-2(0)} = -2$

$2C_1 + C_2 - 4 = -2$

$2C_1 + C_2 = 2$

And $C_1 = -1$

$\therefore -2 + C_2 = 2$

$C_2 = 4$

$\therefore$ ~~$y = e^{2x} (-\cos x e^{2} - e^{2x} \cos x$~~

$\therefore y = e^{2x}(-\cos x + 4\sin x) + 2e^{-2x}$

07 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

Assuming homogeneity, let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$3k^2 e^{kx} - 2ke^{kx} - e^{kx} = 0$

$3k^2 - 2k - 1 = 0$

$k^2 - \frac{2}{3}k - \frac{1}{3} = 0$

$k^2 - \frac{2}{3}k = \frac{1}{3}$ (completing the square)

$k^2 - \frac{2}{3} + \left(-\frac{2}{6}\right)^2 = \frac{1}{3} + \frac{4}{36}$

~~$\left(k - \frac{1}{3}\right)$~~ $= \left(k - \frac{1}{3}\right)^2 = \frac{4}{9}$

$k - \frac{1}{3} = \pm\sqrt{\frac{4}{9}}$

$\therefore k = \pm\frac{2}{3} + \frac{1}{3}$

$k_1 = \frac{2}{3} + \frac{1}{3} = 1$, $k_2 = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$.

But $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$y_{homo} = C_1 e^{x} + C_2 e^{-\frac{1}{3}x}$

Let $y_{non} = Ax + B$

$y' = A$, $y'' = 0$

$\therefore -2A - Ax - B = 2x - 3$

$\therefore -2A - B = -3 \longrightarrow (i)$

$-A = 2 \longrightarrow (ii)$, $A = -2 \longrightarrow (ii)$

$-2(-2) - B = -3$

$4 - B = -3$

$\therefore B = 7$

$\therefore y_{non} = -2x + 7$

but $y = y_{homogenous} + y_{non\text{-}homogenous}$

$\therefore \; y = C_1 e^{x} + C_2 e^{-\frac{1}{3}x} + 7 - 2x$.

08. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Assuming homogenity, let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

Hence,

$k^2 - 6k + 8 = 0$

By completing the squares,

$k^2 - 6k + (-3)^2 = -8 + 9$

$(k-3)^2 = 1$

$k - 3 = \pm\sqrt{1}$

$\therefore k = \pm 1 + 3$

$\therefore k_1 = 4, \quad k_2 = 2$

but $y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$

$\therefore y_{homo} = C_1 e^{4x} + C_2 e^{2x}$

Let $y_{non} = \cancel{Ae^{4x}} \; Axe^{4x}$

$y' = A(4xe^{4x} + e^{4x})$

$y'' = A(\cancel{16e^{4x}} \; 16xe^{4x} + 4e^{4x} + 4e^{4x})$

$= A(16xe^{4x} + 8e^{4x})$

$\therefore \cancel{16Axe^{4x}} + 8Ae^{4x} - \cancel{24Axe^{4x}} - 6Ae^{4x} + \cancel{8Axe^{4x}} = 8e^{4x}$

$2Ae^{4x} = 8e^{4x}$

Dividing both sides by $2e^{4x}$

$\therefore A = 4$

Hence, $y_{non} = 4xe^{4x}$

but $y = y_{homo} + y_{non}$

$\therefore y = C_1 e^{4x} + C_2 e^{2x} + 4xe^{4x}$.