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16/MHS05/030
CIVIL ENGINEERING

Assignment

The parametric equations of a curve are as given in Equations (1) and (2)

$$x = \cos t + \sin t$$

$$y = \sin t - t\cos t$$

In terms of t, determine

(i) an expression for the radius of curvature (R) and

(ii) expressions for the coordinates (h, k) of the centre of curvature

Solutions

(i) Radius of curvature $= \frac{(1 + dy/dx)^{3/2}}{d^2y/dx^2}$

$$R = \frac{(1+\tan^2 t)^{3/2}}{\sec^2 t}$$

$$t\cos t$$

$$R = \frac{(\sec^2 t)^{3/2}}{\sec^2 t}$$

$$t\cos t$$

$$R = \sec^3 t \div \sec^2 t$$

$$t\cos t$$

$$R = \frac{\sec^3 t \times t\cos t}{\sec^2 t}$$

$$R = \sec t \times t\cos t$$

Recall $\sec t = \frac{1}{\cos t}$

$$R = \frac{1}{\cos t} \times t\cos t$$

$$R = t_{11}$$

$x = \cos t + t \sin t$

$y = \sin t - t \cos t$

$\frac{dx}{dt} = -\sin t + \sin t + t\cos t$

$\frac{dx}{dt} = t\cos t \qquad \frac{dt}{dx} = \frac{1}{t\cos t}$

$\frac{dy}{dt} = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = t\sin t \times \frac{1}{\cos t}$

$\frac{dy}{dx} = \frac{t\sin t}{t\cos t}$

$\frac{dy}{dx} = \tan t$

$\frac{d^2y}{dx^2} = \sec^2 t \frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t\cos t} \qquad \frac{-\sec^2 t}{t\cos t}$

$h = x_1 - R\sin t$

$k = y_1 + R\cos t$

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

Coordinate $(\cos t, \sin t)$.