Nnamah Ifeoma Olive
Computer Engineering
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① $x = \cos t + t\sin t$

$y = \sin t + t\cos t$

Using $U\frac{dv}{dx} + V\frac{du}{dx}$ where $u = t$, $v = \sin t$

$\frac{d(t\sin t)}{dt} = t(\cos t) + \sin t(1)$ $\quad \frac{du}{dt} = 1$, $\frac{dv}{dt} = \cos t$

$= t\cos t + \sin t$

$\frac{d(\cos t)}{dt} = -\sin t$

$\frac{dx}{dt} = \frac{d(\cos t)}{dt} + \frac{d(t\sin t)}{dt}$

$= -\sin t + (t\cos t + \sin t)$

$= -\cancel{\sin t} + t\cos t + \cancel{\sin t}$

$\frac{dx}{dt} = t\cos t$

where, $u = t$, $v = \cos t$

$\frac{du}{dt} = 1$, $\frac{dv}{dt} = \sin t$

$\frac{d(t\cos t)}{dt} = t(-\sin t) + \cos t(1)$

$= -t\sin t + \cos t$

$\frac{d(\sin t)}{dt} = \cos t$

$\frac{dy}{dt} = \frac{d(\sin t)}{dt} - \frac{d(t\cos t)}{dt}$

$= \cos t - (-t\sin t + \cos t)$

$= \cancel{\cos t} + t\sin t - \cancel{\cos t}$

$= t\sin t$

since $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{dy}{dx} = t\sin t \times \frac{1}{t\cos t}$$

$$= \frac{\cancel{t}\sin t}{\cancel{t}\cos t}$$

$$= \frac{\sin t}{\cos t}$$

$$\frac{dy}{dx} = \tan t$$

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \times \frac{dt}{dx}$$

$$= \sec^2 t \times \frac{1}{t\cos t}$$

since $\frac{1}{\cos t} = \sec t$

$$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t} \times \frac{1}{\cos t}$$

$$= \frac{\sec^2 t \times \sec t}{t}$$

$$= \frac{\sec^3 t}{t}$$

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left(1 + (\tan t)^2\right)^{3/2}}{\frac{\sec^3 t}{t}}$$

$$= \frac{\left(1^2 + \tan^2 t\right)^{3/2}}{\sec^3 t} t$$

$$= \frac{\left(1 + \tan^2 t\right)^{3/2}}{\sec^3 t} t$$

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Recall $1 + \tan t = \sec t$

$$R = \frac{(\sec t)^{\cancel{2} \times 3/\cancel{2}}\, t}{\sec^3 t}$$

$$= \frac{\cancel{(\sec t)^3}\, t}{\cancel{\sec^3 t}}$$

$R = t$ units

Centre of curvature

$h = x_1 - R\sin\theta$

$k = y_1 + R\cos\theta$

since $\tan\theta = \frac{dy}{dx}$

$\therefore \tan\theta = \tan t$

$x_1 = \cos t + t\sin t$

$h = \cos t + \cancel{t\sin t} - \cancel{t\sin t}$

$= \cos t$

$y_1 = \sin t - t\cos t$

$k = \sin t - \cancel{t\cos t} + \cancel{t\cos t}$

$= \sin t$

$\therefore$ The centre of curvature for the parametric equ. is $(h, k) = (\cos t, \sin t)$