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16 MHS 03 030
CIVIL ENGINEERING
GNG 281

Assignment

1 a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right]$

Soln

Let $u = x^2 - \pi/4$ & $v = \sin(\cos x)$ Numerator

then

$dy/dx = u\frac{dv}{dx} + v\frac{du}{dx}$

For $u = x^2 - \pi/4$

$\frac{du}{dx} = 2x$

For $v = \sin(\cos x)$

Let $t = \cos x$ and $v = \sin t$

$\frac{dt}{dx} = -\sin x \qquad \frac{dv}{dt} = \cos t$

$\frac{dv}{dx} = \frac{dt}{dx} \times \frac{dv}{dt} = -\sin x \cos t$

$\frac{dv}{dx} = -\sin(\cos(\cos x))$

$\frac{dy}{dx} = [x^2 - \pi/4] - \cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$

$\frac{dy}{dx} = [x^2 - \pi/4] . - \cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$

Denominator

Let $a = x - \pi/2$

$da/dx = 1$

$\therefore \frac{(x^2 - \pi/4)(-\cos(\cos x)\sin x) + \sin(\cos x) 2x}{1}$

$\lim_{x \to \pi/2} (\frac{\pi^2}{2} - \pi/4)(-\cos(\cos \pi/2)\sin \pi/2) + \sin(\cos \pi/2) 2\pi/2$

$= (\pi^2/4 - \pi/4)(-1) + 0$

$= \left(\frac{\pi^2 - \pi}{4}\right) - 1 = \frac{-\pi^2 + \pi}{4}$

$= \frac{\pi(-\pi+1)}{4}$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2}\right] = \frac{\pi(-\pi+1)}{4}$

b. $\lim_{x \to \pi/2} \ln\left[\frac{\exp(3x^2 + 2x - 1)}{x+1}\right]$

Soln

$\ln\left[\frac{\exp(3(\pi/2)^2 + 2(\pi/2) - 1)}{\pi/2 + 1}\right]$

$\ln\left[\frac{\exp(3(\pi^2/4) + \cancel{2}(\pi/\cancel{2}) - 1)}{\pi/2 + 1}\right]$

$= \frac{\frac{3\pi^2}{4} + \pi - 1}{\frac{\pi + 2}{2}} = \frac{3\pi^2 + 4\pi - 4}{4} \times \frac{2}{\pi + 2}$

$= \frac{3\pi^2 + 4\pi - 4}{2(\pi + 2)} = \frac{(\pi/1 - 2/3)(\pi + 2)}{2(\pi + 2)}$

$= \frac{3\pi - 2}{3} \times \frac{1}{2} = \frac{3\pi - 2}{6}$

$\therefore \lim_{x \to \pi/2} \ln\left[\frac{\exp(3x^2 + 2x - 1)}{x+1}\right] = \frac{3\pi - 2}{6}$

c. $\lim_{x \to 2\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

$= \cos\left[\sin^{-1}\frac{(2\sqrt{3}-2)}{2\sqrt{3}-\sqrt{3})}\right]$

$= \cos\left[\sin^{-1}\sqrt{3}/2\right]$

$= \cos 60 = 1/2$

$\therefore \lim_{x \to 2\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right] = 1/2$

d. $\lim_{x \to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Soln

$\lim_{x \to 4}\left[\frac{2x-8}{2x-5}\right]$

$= \frac{2(4)-8}{2(4)-5} = 0/3 = 0$

$\therefore \lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = 0$

No 2

a. $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots\cdots$

Soln

If we take our standard series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \cdots\cdots$

When $p = 2$ we get

$\frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \frac{2}{5^2} + \cdots\cdots$

Since

$\frac{2}{2\times3} < \frac{2}{2^2}$; $\frac{2}{3\times4} < \frac{2}{3^2}$; $\frac{2}{4\times5} < \frac{2}{4^2}$

There fore, the given series is convergent

b. $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Soln

using standard series when $p = 2$

$\frac{2}{1^p} + \frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \cdots$

Since $p > 1$ the series converges

Therefore, the given series is convergent

c. (a) $U_n = \frac{1+2n^2}{1+m}$

Soln

$\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{\left[1/n^2 + 2n^2/n^2\right]}{1/n^2 + n^2/n^2}$

$= \left[\frac{1/n^2 + 2/1}{1/n^2 + 1/1}\right]$

$= \frac{0+2}{0+1}$

$= 2$

$\lim_{n \to \infty} U_n = 2$

Since $\lim_{n \to \infty} U_n \neq 0$

Therefore, the series is divergent

No 3

$\frac{x}{27} + \frac{x^2}{125} + \cdots\cdots + \frac{x^n}{(2n+1)^3}$

Soln

$$|u_n| = \frac{x^n}{(2n+1)^3}$$

$$|u_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{|u_{n+1}|}{|u_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x^n \cdot x}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

$$= \frac{x\left[\frac{8n^3}{n^3} + \frac{12n^2}{n^3} + \frac{6n}{n^2} + \frac{1}{n^3}\right]}{\frac{8n^3}{n^3} + \frac{36n^2}{n^2} + \frac{54n}{n^2} + \frac{27}{n^3}}$$

$$= \lim_{n\to\infty} \frac{x\left[8 + 12/n + 6/n^2 + 1/n^3\right]}{8 + 36/n + 54/n^2 + 27/n^3}$$

$$\frac{x[8+0+0+0]}{8+0+0+0}$$

$$8x/8 = x.$$

4. Evaluate using L'Hospital's Rule.

$$\lim_{x\to 0}\left[\frac{\sin x - \cos x}{x^3}\right]$$

Soln

$$\lim_{x\to 0}\left[\frac{\sin x - \cos x}{x^3}\right]$$

$$\lim_{x\to 0}\left[\frac{\cos x + \sin x}{3x^2}\right]$$

$$\lim_{x\to 0}\left[\frac{\sin x + \cos x}{6x}\right]$$

$$\lim_{x\to 0}\left[\frac{-\cos x + (-\sin x)}{6}\right]$$

$$\lim_{x\to 0}\left[\frac{-\cos x - \sin x}{6}\right]$$

$$\frac{-1+0}{6} = -1/6$$

$$\lim_{x\to 0}\left[\frac{\sin x - \cos x}{x^3}\right] = -1/6.$$