Nwosu Ikedinachi
Mechanical Eng
16/ENG06/043

1.) The parametric equation of a curve are as given in equation 1 and 2

$$x = \cos t + t\sin t - 1$$
$$y = \sin t - t\cos t - 2$$

In terms of t determine

a.) An expression for the radius of curvature

Solution

$$R \Rightarrow \frac{\left(1 + \left[\frac{dy}{dx}\right]^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$x \Rightarrow \cos t + t\sin t$$
$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$$
$$dx/dt = t\cos t$$

$$y = \sin t - t\cos t$$
$$dy/dt = \cos t - (-t\sin t + \cos t)$$
$$= \cos t + t\sin t - \cos t$$
$$= t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$= t\sin t \times \frac{1}{t\cos t} = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$
$$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$
$$= \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \text{given}$$

where $v = \cos t$

$dv = \sin t$

$u = \sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

Recall that $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

Also recall that

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$R = \frac{\left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2}}{1/\cos^3 t}$$

$$R = \left(1 + \frac{\sin^2 t}{\cos^2 t}\right)^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right) \times t\cos^3 t$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t = \frac{1}{(\sqrt{\cos^2 t})^3} \times t\cos^3 t$$

$$R = \frac{t\cos^3 t}{\cos^3 t} = t$$

$\therefore$ The radius of curvature is t

2) Expressions for the co-ordinates (h, k) of the centre of curvature

$\Rightarrow h = x_1 - R\sin\theta$ -1

$k = y_1 + R\cos\theta$ -2

$R = t \quad t = \theta$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

Substituting x_1 in eqn 1 and y_1 in eqn 2

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

The expressions for the coordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$