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ENG 281 Assignment

The parametric equation of a curve are as given in Equation 1 and 2:

$$x = \cos t + t\sin t$$

$$y = \sin t - t\cos t$$

In terms of t, determine:

i) an expression for the radius of curvature (R), and

ii) expressions for the coordinates (h, k) of the centre of curvature.

Solution

$x = \cos t + t\sin t$ $\qquad y = \sin t - t\cos t$

$$\frac{dx}{dt} = \frac{d\cos t}{dt} + \frac{d\,t\sin t}{dt}$$

$$\frac{d\cos t}{dt} = -\sin t$$

for $\frac{d\,t\sin t}{dt}$ let $t = u$ and $v = \sin t$

$$\frac{du}{dt} = 1 \quad \text{and} \quad \frac{dv}{dt} = \cos t$$

$$\Rightarrow v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= \sin t(1) + t(\cos t)$$

$$= \sin t + t\cos t$$

$$\frac{dx}{dt} = -\sin t + (\sin t + t\cos t)$$

$$= \cancel{-\sin t} + \cancel{\sin t} + t\cos t$$

$$= t\cos t$$

$y = \sin t - t\cos t$

$$\frac{dy}{dt} = \frac{d\sin t}{dt} - \frac{d\,t\cos t}{dt}$$

$$\frac{d\sin t}{dt} = \cos t$$

for $\frac{d\,t\cos t}{dt}$ $\qquad t\cos t$

let $u = t$ and $v = \cos t$

$$\frac{du}{dt} = 1 \quad \text{and} \quad \frac{dv}{dt} = -\sin t$$

$$v\frac{du}{dt} + u\frac{dv}{dt} = \cos t(1) + t(-\sin t)$$

$$= \cos t - t\sin t$$

$$\frac{dy}{dt} = \cos t - (\cos t - t\sin t)$$

$$= \cancel{\cos t} - \cancel{\cos t} - -t\sin t$$

$$= t\sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dt} = mt\sin t$$

$$\frac{dt}{dx} = \frac{1}{t\cos t}$$

$$\frac{dy}{dx} = mt\sin t \times \frac{1}{t\cos t}$$

$$= \frac{mt\sin t}{t\cos t} = m\tan t$$

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \cdot \frac{dt}{dx}$$

$$= \sec^2 t \cdot \frac{1}{t\cos t}$$

$$= \frac{\sec^2 t}{t\cos t}$$

$$\frac{1}{\cos t} = \sec t$$

$$= \frac{\sec^2 t \times \sec t}{t} = \frac{\sec^3 t}{t} = t^{-1}\sec^3 t$$

$$R = \frac{\left(1+(dy/dx)^2\right)^{3/2}}{d^2y/dx^2}$$

$$= \frac{\left(1+(\tan t)^2\right)^{3/2}}{t^{-1}\sec^3 t}$$

$$= \frac{\left(1+\tan^2 t\right)^{3/2}}{t^{-1}\sec^3 t} = \frac{(\sec t)^{2\times 3/2}}{t^{-1}\sec^3 t}$$

since $1+\tan^2 t = \sec^2 t$

$$= \frac{(\sec t)^3}{t^{-1}(\sec t)^3} = \frac{1}{t^{-1}} = t \text{ units.}$$

For

Centre of Curvature.

$h = x_1 - R\sin\theta$ $\qquad$ $h = x_1 - R\sin t$

$k = y_1 + R\cos\theta$ $\qquad$ $k = y_1 + R\cos t$

$$\tan\theta = \frac{dy}{dx}$$

$$\tan\theta = \tan t$$

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$x_1 = \cos t + t \sin t$

$h = \cos t + t \sin t - R \sin t$

$h = \cos t + t \sin t - t \sin t$

$= \cos t \qquad R = t$

$y_1 = \sin t - t \cos t$

$K = \sin t - t \cos t + R \cos t$

$K = \sin t - t \cos t + t \cos t$

$= \sin t$

Centre of curvature for the parametric equation is therefore

$(h, k) = (\cos t, \sin t)$.

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