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COURSE: ENG 281 (Engineering maths)
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Assignment.

1. Evaluate the limits of the functions:

a. $\lim \left[\frac{(x^2 - \frac{\pi}{4}) \sin(\cos x)}{x - \frac{\pi}{2}}\right]$

Solution.

let $u = x^2 - \pi/4$. (for numerator)

and $v = \sin(\cos x)$.

$\frac{du}{dx} = 2x$

$\frac{dv}{dx}$ for $v = \sin(\cos x)$

let $a = \cos x$

$\frac{da}{dx} = -\sin x$.

$v = \sin a$

$\frac{dv}{da} = \cos a$

$\frac{dv}{da} \times \frac{da}{dx} = -\sin x \cos a$.

$\frac{dv}{dx} = -\sin x \cos(\cos x) = -\sin x \cos(\cos x)$

$u \frac{dv}{dx} + v \frac{du}{dx}$

$= ((x^2 - \pi/4)(-\sin x \cos(\cos x)) + (\sin(\cos x) \cdot 2x)$

For denominator,

$y = x - \frac{\pi}{2}$

$dy/dx = 1$

$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin \cos(x}{x - \pi/2}\right] = \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(-\sin x \cos(\cos x)) + (\sin(\cos x) \cdot 2x)}{1}\right]$

$= (\frac{\pi}{2})^2 - \frac{\pi}{4} \cdot (-\sin\frac{\pi}{2} \cos(\cos\frac{\pi}{2}) + (\sin(\cos\frac{\pi}{2}) \cdot 2x]$

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \cdot 1 + 0$

$= \frac{\pi^2}{4} - \frac{\pi}{4}$

b. $\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x+1}\right)\right]$ $\exp = 1/\ln$

$\lim_{x \to \pi/2} \ln\left[\frac{1}{\ln} \frac{(3x-1)(x+1)}{(x+1)}\right]$

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$$\lim_{x \to \pi/2} [3x - 1)$$

$= 3x - 1$

$= 3\left(\frac{\pi}{2}\right) - 1$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1} \frac{(x-2)}{(x-\sqrt{3})} \right)$

$\cos\left(\sin^{-1} \frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})} \right)$

$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2} \right) \right]$

$= \cos 60$

$= 1/2$

d) $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$= \frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4}$

$= \frac{16 - 32 + 16}{16 - 20 + 4} = \frac{0}{0} = 0$

differentiating:

$\lim_{x \to 4} \left[\frac{2x - 8}{2x - 5} \right]$

$\frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0$

2) Determine whether each of the following series is convergent!

a. $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \ldots$

$= \frac{2}{6} + \frac{2}{12} + \frac{2}{20} + \frac{2}{30} + \ldots$

$r = \frac{\cancel{2}}{12} \times \frac{6}{\cancel{2}}$ $r = 1/2$

$a = 2/6 = 1/3$

$S_n = \frac{a(1 - r^n)}{1 - r}$ where $a = 1/3$ and $r = 1/2$ and $n = \infty$

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$$S_n = \frac{1/3(1 - 1/2^n)}{1 - 1/2}$$

$n \to \infty$, $1/2^n \to 0$

$$S_n = \frac{1/3(1-0)}{1-1/2}$$

$S_n = 1/3 \times 2/1$

$= 2/3$

$\therefore$ The series converges since $2/3$ is a definite value

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots$

$\frac{2}{1} + \frac{2}{4} + \frac{2}{9} + \frac{2}{16} + \ldots$

$r = \frac{2}{4} \div \frac{2}{1}$

$= \frac{2}{4} \times 1/2$

$r = 1/4$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$a = 2$, $r = 1/4$, $n = \infty$

$$S_n = \frac{2(1-1/4^n)}{1-1/4}$$

as $n \to \infty$, $1/4 \to 0$

$$S_n = \frac{2(1-0)}{3/4}$$

$$S_n = \frac{2}{3/4}$$

$S_n = 8/3$

$\therefore$ The series converges since $8/3$ is a definite value.

c) $U_n = \frac{1+2n^2}{1+n^2}$.

$\lim_{n\to\infty} U_n = \frac{1+2n^2}{1+n^2}$.

$\therefore U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2} = \frac{1/n^2 + 2}{1/n^2 + 1}$

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as $n \to \infty$, $\frac{1}{n^2} = 0$

$U_n = \frac{0+2}{0+1}$

$U_n = 2$.

$\therefore$ The series is divergent since $U_n \neq 0$

3) $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$

$U_{n+1} = \frac{x^{n+1}}{(2n+1)^3 + 1}$

$U_{n+1} = \frac{x^{n+1}}{[2(n+2)+1]}$

$U_{n+1} = \frac{x^{n+1}}{(2n+3)^3}$

$U_{n+1} = \frac{x^{n+1}}{8n^3 + 36n^2 + 54n + 27} \div \frac{x^n}{(2n+1)^3}$

$= \frac{x^{n+1}}{8n^3+36n^2+54n+27} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x^n \cdot x^1}{8n^3+36n^2+54n+27} \times \frac{8n^3+12n^2+6n+1}{x^n} = x \left[\times \frac{8n^3+12n^2+6n+1}{8n^3+36n^2+54n+27} \right.$

$= x \left[\frac{8n^3+12n^2+16n+1}{8n^3+36n^2+54n+27} \right] = \lim_{n\to\infty} x \left[\frac{8 + 12/n + 6/n^2}{8 + 36/n + 54/n^2 + 27/n^3} \right]$

$= x \left[\frac{8+0+0}{8+0+0+0} \right]$

$\frac{U_{n+1}}{U_n} = x$. $\quad -1 \leq x < 1$ (it converges)

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4) $\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$

$= \frac{\sin 0 - \cos 0}{0^3}$

$= \frac{0 - 1}{0} = \frac{-1}{0}$

differentiating

$\frac{\cos x + \sin x}{3x^2}$

$= \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1+0}{0}$ = undefined.

differentiating again.

$\frac{-\sin x + \cos x}{6x} = \frac{0+1}{0}$ = undefined

differentiating again

$\frac{-\sin x + \cos x}{6x} = \frac{-\cos x - \sin x}{6}$

$= \frac{-\cos 0 - \sin 0}{6} = \frac{-1 - 0}{6} = {}^{-1}/_{6}$.