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1a $\lim_{x \to \frac{\pi}{2}} \left[\frac{\left(x^2 - \frac{\pi}{4}\right) \sin(\cos x)}{x - \frac{\pi}{2}} \right]$

Solution
numerator

$u = x^2 - \frac{\pi}{4}$, $V = \sin(\cos x)$

$\frac{du}{dx} = 2x$, $\frac{dV}{dx} = -\sin x \cos(\cos x)$

let

$a = \cos x$, $\frac{da}{dx} = -\sin x$

$v = \sin a$, $\frac{dv}{da} = \cos a$

$$\frac{dV}{dx} = \frac{dV}{da} \cdot \frac{da}{dx}$$

$$\frac{dv}{dx} = \cos a \cdot (-\sin x)$$

$$\frac{dv}{dx} = -\sin x \cos a = -\sin x \cos(\cos x)$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = \left(x^2 - \frac{\pi}{4}\right)(-\sin x \cos(\cos x)) + \sin(\cos x)(2x)$$

$$\frac{dy}{dx} = \left(x^2 - \frac{\pi}{4}\right)(-\cos(\cos x)\sin x) + \sin(\cos x)(2x)$$

denominator

let $m = x - \frac{\pi}{2}$, $\frac{dm}{dx} = 1$

$$\lim_{x \to \frac{\pi}{2}} \left[\frac{\left(x^2 - \frac{\pi}{4}\right) - \cos x(\cos x)\sin x + \sin(\cos x)(2x)}{1} \right]$$

$$= \frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) - \cos\left(\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + \sin\left(\cos\frac{\pi}{2}\right)2\left(\frac{\pi}{2}\right)}{1}$$

$$= \frac{\left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)(-1) + 0}{1} = -\frac{\pi^2}{4} + \frac{\pi}{4}$$

$$= \frac{-\pi^2 + \pi}{4}$$

$$\therefore \lim_{x \to \frac{\pi}{2}} \left[\frac{\left(x^2 - \frac{\pi}{4}\right)(\sin(\cos x))}{x - \pi/2} \right] = -\frac{\pi^2 + \pi}{4}$$

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b $\lim_{x \to \frac{\pi}{2}} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right]$

solution

$\lim_{x \to \frac{\pi}{2}} \cancel{\ln} \left[\cancel{\exp} \frac{(3x^2 + 2x - 1)}{x+1} \right]$

$= \dfrac{3\left(\frac{\pi}{2}\right)^2 + \cancel{2}\left(\frac{\pi}{\cancel{2}}\right) - 1}{\frac{\pi}{2} + 1}$

$= \dfrac{3\frac{\pi^2}{4} + \pi - 1}{\frac{\pi}{2} + 1}$

$= \dfrac{3\pi^2}{4} + \dfrac{\pi}{1} - \dfrac{1}{1} \div \dfrac{\pi + 2}{2}$

$= \dfrac{3\pi^2 + 4\pi - 4}{\cancel{4}\,2} \times \dfrac{\cancel{2}}{\pi + 2}$

$= \dfrac{3\pi^2 + 4\pi - 4}{2(\pi + 2)}$

$= \dfrac{(3\pi - 2)\cancel{(\pi + 2)}}{2\cancel{(\pi + 2)}}$

$= \dfrac{3\pi - 2}{2}$

$= \dfrac{3\pi}{2} - \dfrac{2}{2}$

$= \dfrac{3\pi}{2} - 1$

$\therefore \lim_{x \to \frac{\pi}{2}} \ln \left[\exp \frac{(3x^2 + 2x - 1)}{x+1} \right] = \dfrac{3\pi}{2} - 1$

c $\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right]$

solution

$\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x+2)}{(x-\sqrt{3})}\right]$

$= \cos\left[\frac{\sin^{-1}(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \cos\left[\sin^{-1}(0.8660)\right]$

$= \cos 60$

$= \frac{1}{2}$

$\therefore \lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$

d $\lim_{x \to 4} \left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Solution

$= \frac{4^2-8(4)+16}{4^2-5(4)+4}$

$= \frac{16-32+16}{16-20+4}$

$= \frac{0}{0}$ (undefined)

$\lim_{x \to 4} \frac{f'(x)}{g''(x)} \left(\frac{2x-8}{2x-5}\right) = \frac{2(4)-8}{2(4)-5}$

$= \frac{0}{3}$

$= 0$

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2a) $U_n = \frac{2}{(n+1)(n+2)}$

$U_{n+1} = \frac{2}{(n+1+1)(n+2+1)}$

$= \frac{2}{(n+2)(n+3)}$

$\frac{U_{n+1}}{U_n} = \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \cdot \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$

$= \frac{n+1}{n+3}$

$\lim_{n \to \infty} \left(\frac{n+1}{n+3}\right) = \left(\frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n} + \frac{3}{n}}\right)$

$= \frac{1+0}{1+0}$

$= \frac{1}{1}$

$= 1$

Since $\frac{U_{n+1}}{U_n} = 1$, $\therefore$ the series is either Convergent or divergent

b) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

Since $P > 1$ the series converges

c) $U_n = \frac{1+2n^2}{1+n^2}$

$\lim_{n \to \infty} \left(\frac{1+2n^2}{1+n^2}\right) = \left(\frac{\frac{1}{n^2} + \frac{2\cancel{n^2}}{\cancel{n^2}}}{\frac{1}{n^2} + \frac{\cancel{n^2}}{\cancel{n^2}}}\right)$

$= \left(\frac{0+2}{0+1}\right) = 2$

Since $\lim_{n \to \infty} U_n \neq 0$ the series divergence

3 $u_n = \frac{x^n}{(2n+1)^3}$

$u_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3}$

$= \frac{x^{n+1}}{(2n+3)^3}$

$\frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{\cancel{x^n} \cdot x^1}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$

$= \frac{x(2n+1)^3}{(2n+3)^3}$

$\lim_{n \to \infty} \left(\frac{x(2n+1)^3}{(2n+3)^3} \right)$

$\lim_{n \to \infty} x \left(\frac{8n^3+12n^2+6n+1}{8n^3+36n^2+54+27} \right)$

$\lim_{n \to \infty} \left(\frac{8xn^3+12xn^2+6xn+x}{8n^3+36n^2+54n+27} \right)$

$= \left(\frac{\frac{8x\cancel{n^3}}{\cancel{n^3}} + \frac{12x\cancel{n^2}}{n^{\cancel{3}}} + \frac{6x\cancel{n}}{n^{\cancel{3}2}} + \frac{x}{n^3}}{\frac{8\cancel{n^3}}{\cancel{n^3}} + \frac{36\cancel{n^2}}{n^{\cancel{3}}} + \frac{54\cancel{n}}{n^{\cancel{3}2}} + \frac{27}{n^3}} \right)$

$= \left(\frac{8x + \frac{12x}{n} + \frac{6x}{n^2} + \frac{x}{n^3}}{8 + \frac{36}{n} + \frac{54}{n^2} + \frac{27}{n^3}} \right)$

$= \frac{8x+0+0+0}{8+0+0+0}$

$= \frac{\cancel{8}x}{\cancel{8}}$

$\therefore \lim_{n \to \infty} \frac{x(2n+1)^3}{(2n+3)^3} = x$

Since $-1 \leq x \leq 1$

the series is convergent.

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4 $\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

Solution

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) = \frac{\sin 0 - \cos 0}{0^3}$$

$$= \frac{-1}{0} \text{ (undefined)}$$

$$\lim_{x \to 0} \frac{f'(x)}{g'(x)} \left(\frac{\cos x + \sin x}{3x^2} \right)$$

$$= \frac{\cos 0 + \sin 0}{3(0)^2}$$

$$= \frac{1}{0} \text{ (undefined)}$$

$$\lim_{x \to 0} \frac{f''(x)}{g''(x)} \left(\frac{-\sin x + \cos x}{6x} \right)$$

$$= \frac{-\sin 0 + \cos 0}{6(0)}$$

$$= \frac{1}{0} \text{ (undefined)}$$

$$\lim_{x \to 0} \frac{f'''(x)}{g'(x)} \left(\frac{-\cos x - \sin x}{6} \right)$$

$$= \frac{-\cos 0 - \sin 0}{6}$$

$$= \frac{-1}{6}$$

$$\therefore \lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) = \frac{-1}{6}$$