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The parametric equation of curve are as given in equation (1) and (2)

$$x = \cos t + t \sin t$$

$$y = \sin t - t \cos t$$

In terms of t determine.

i. An expression for the radius of curvature (R) and

ii. Expression for the coordinates of the centre of curvature

Solution

Recall

$$\Rightarrow R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$x = \cos t + t \sin t$

$y = \sin t - t \cos t$

$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t \ (1)$$

$$\frac{dx}{dt} = t\cos t$$

$$\frac{dy}{dt} = \cos t + t\sin t - \cos t \ (1)$$

$$\frac{dy}{dt} = t\sin t$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\sin t}{t\cos t}\right) \cdot \frac{dt}{dx}$$

$$\frac{V\frac{du}{dx} - U\frac{dv}{dx}}{} = \frac{V\frac{du}{dt} - U\frac{dv}{dt}}{V^2}$$

Let $U = \sin t$

$V = \cos t$

$\frac{du}{dt} = \cos t$

$\frac{dv}{dt} = -\sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t - (-\sin t)}{\cos^2 t} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

Recall from trigonometric identities

that $\sin^2\theta + \cos^2\theta = 1$

$$\frac{d^2y}{dx} = \frac{1}{t\cos^3 t}$$

Since $R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)\right]^{3/2} \div \frac{d^2y}{dx^2}$$

$$R = \left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right] \times \frac{t\cos^3 t}{1}$$

$R = \dfrac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$

$R = \dfrac{t\cos^2 t}{\cos^2 t}$

$R = t$

$\therefore$ The expression for the radius of curvature is t.

ii. (h, k)

Recall $h = x_1 - R\sin\theta \ldots (1)$

$k = y_1 + R\cos\theta \ldots (2)$

$R = t \quad \theta = t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

Substituting for θ, x_1, y_1 and R - equation (1) and (2)

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

Hence, the expression of the coordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$.