AGBEDE GOODNESS YERINMINI
15/ENG03/003
CIVIL ENGINEERING
ENG 381 (ENGINEERING MATHS)

1. $\frac{d^2y}{dx^2} - \frac{2y}{2x} - 2y = 8$

$m^2 - m - 2 = 0$

$(m^2 - 2m)(m - 2) = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1$ and 2.

$y = Ae^{2x} + Be^{-x}$

$y = C$

$\frac{dy}{dx} = 0$

$\frac{dy}{dx^2} = 0$.

$\frac{d^2y}{dx^2} - \frac{2y}{dx} - 2y = 8$

$0 - 0 - 2C = 8$

$C = -4$

P.I $y = -4$ //

$y = Ae^{2x} + Be^{-x} - 4$.

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = \pm 4$

$m = \pm\sqrt{4}$

$m = \pm 2$.

C.I $= A\cosh 2x + B\sinh 2x$

P.I $= y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{2x^2} = 9Ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 2$

$y = 2e^{3x}$

$G.S = Ae^{2x} + Be^{-2x} + 2e^{3x}$

3. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$(m^2 + m) + (m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ Twice

$y = e^{-x}(A + Bx)$.

$P.I = y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$= 4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$y = e^{-2x}$

$G.S = e^{-x}(A + Bx) + e^{-2x}$

4. $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m = \pm\sqrt{25}$

$m = \pm 5$

$C.I = A\cos 5x + B\sin 5x$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E + 25Cx^2 + 25Dx = 5x^2 + x$

$25C = 5$

$C = \frac{5}{25} = \frac{1}{5}$

$25D = 1$

$D = 1/25$

$2C + 25E = 0$

$2C(1/5) + 25E = 0$

$2/5 = -25E$

$E = -2/125$

$y = 1/5\,x^2 + 1/25\,x - 2/125$

$C.S = A\cos 5x + B\sin 5x + 1/5\,(x^2 + 1/5\,x - 2/25)$

5- $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$.

$y = C\cos x + D\sin x)$

$\frac{dy}{dx} = -\sin x + Dx\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$= -C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$= 2C\sin x - 2D\cos x = 4\sin x$

Compare both eqn

$2C = 4$

$C = 2$

$-2D = 0$

$D=0$

$y = (C\cos x + D\sin x)$

$y = 2\cos x + 0 = 2\cos x$

$G.S = e^x(A+Bx) + 2\cos x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x=0, y=1$ and $dy/dx = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2-4\times1\times5}}{2\times1}$

$= \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm j\sqrt{4}}{2}$

$m = \frac{-4}{2} \pm j\frac{\sqrt{4}}{2} = -2 \pm j$

$\alpha = -2 \quad \beta = 1$

$CF = y = e^{-2x}(A\cos x + B\sin x)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} - 2e^{-2x}(4) + 5Ce^{-2x} = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$y = 2e^{-2x}$

$G.S = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

given that $x=0$ $y=1$ and $\frac{dy}{dx} = -2$

$y = Ae^{-2x}\cos x + Be^{-2x}\sin x + 2e^{-2x}$

$1 = Ae^{-2(0)}\cos 0 + Be^{-2(0)}\sin 0 + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2 = -1$

$\frac{dy}{dx} = -2Ae^{-2x}\cos x - \sin x Ae^{-2x} - 2Be^{-2x}\sin x + \cos x Be^{-2x} - 4e^{-2x}$

$-2 = -2Ae^{-2(0)} + Be^{-2(0)} - 4e^{-2(0)}$

$-2 = -2A + B - 4$

$-2 = -2(-1) + B - 4$

$-2 = +2 + B - 4 = -2 + 2 = B$

$y = e^{-2x}(-\cos x + 0\sin x) + 2e^{-2x}$.

$y = e^{-2x}\cos x + 2e^{-2x} = e^{-2x}(-\cos x + 2)$.

7. $3\frac{d^2y}{dx^2} - 2\frac{dy}{2f} - y = 2x - 3$.

$3m^2 - 2m - 1 = 0$.

$(3m^2 - 3m)(m - 1) = 0$.

$3m(m-1) + 1(m-1) = 0$.

$(3m+1)(m-1) = 0$

$m = -1/3$ & 1

$y = Ae^{-1/3x} + Be^{x}$

P.I $= y = Cx + D$.

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$.

$3(0) - 2C - (Cx + D) = 2x - 3$.

$-2C - Cx - D = 2x - 3$.

Compare the two eqn.

$-Cx = 2x$

$-2C - D = -3$.

$-2(-2) - D = -3$.

$4 - D = -3$.

$-D = -3 - 4$

$-D = -7$

$D = 7$.

$y = -2x + 7$

G.S $= Ae^{-x/3} + Be^{x} - 2x + 7$

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$(m^2-4m)(-2m+8)=0$

$m(m-4)-2(m-4)=0$

$(m-2)(m-1)=0$

$m = 2$ and 4.

$y = Ae^{2x} + Be^{4x}$

$y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Ce^{4x}$

$= 16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

C is Undefined.

$\therefore y = Cxe^{4x}$

Using product rule to differentiate.

$u = Cx \quad v = e^{4x}$

$du/dx = C \quad dv/dx = 4e^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$.

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = 4$

$y = 4xe^{4x}$

G.S $= Ae^{2x} + Be^{4x} + 4xe^{4x}$