QUESTION 1

(i) $d^2y/d\theta^2 + 4\,dy/d\theta + 5y = 6\sin\theta$

$d^2y/d\theta^2 + 4\,dy/d\theta + 5y = 0$

$m^2 + 4m + 5 = 0$

$a = 1,\ b = 4,\ c = 5$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \dfrac{-4 \pm \sqrt{-4}}{2}$

$m = \dfrac{-4 \pm j2}{2} = -2 \pm j1$

C.F:

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

P.I $\Rightarrow$ $y = C\sin\theta + D\cos\theta$

$dy/d\theta = C\cos\theta - D\sin\theta$

$d^2y/d\theta^2 = -C\sin\theta - D\cos\theta$

$-C\sin\theta - D\cos\theta + 4(C\cos\theta - D\sin\theta) + 5(C\sin\theta + D\cos\theta) = 6\sin\theta$

$\sin\theta(-C - 4D + 5C) = 6\sin\theta$

$\cos\theta(-D + 4C + 5D) = 0$

$4C - 4D = 6$ (i)

$4C + 4D = 0$ (ii)

From (ii)

$C + D = 0$

$C = -D$

$4(-D) - 4D = 6$

$-8D = 6$

$D = -6/8 = -3/4$

$C + D = 0$

$C = -D$

$C = -(-3/4) = 3/4$

P.I $\Rightarrow$ $y = 3/4\sin\theta - 3/4\cos\theta$

G.S $\Rightarrow$ C.F + P.I

$y = e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\sin\theta - 3/4\cos\theta$

(iii)

At steady state, $\theta = \infty$, $dy/d\theta = 0$

$\dfrac{dy}{d\theta} = e^{-2\theta}(-A\sin\theta + B\cos\theta) - 2e^{-2\theta}(A\cos\theta + B\sin\theta) + 3/4\cos\theta + 3/4\sin\theta$

at $\theta = \infty$, $dy/d\theta = 0$

~~$0 = e^{-2\cos\infty}(-A\sin\infty + B\cos\infty) - 2e^{-2\infty}(A\cos\infty + B\sin\infty) + 3/4\cos\infty + 3/4\sin\infty$~~

$0 = 0 - 0 + 3/4\cos\theta + 3/4\sin\theta$

$3/4\cos\theta = -3/4\sin\theta$

$-1 = \dfrac{3/4\sin\theta}{3/4\cos\theta}$

$-1 = \tan\theta$

$\theta = \tan^{-1}(-1)$

$\theta = 135°$

QUESTION 2

$EI\, d^2y/dx^2 = \frac{w}{2}(L-x)^2$

$EI\, d^2y/dx^2 = 0$

$EIm^2 = 0$

$m = 0 \pm 0$

$y = e^{0x}(A + Bx)$

$y = A + Bx$

P.I $\Rightarrow y = Cx^2 + Dx + E$

$dy/dx = 2Cx + D$

$d^2y/dx^2 = 2C$

$EI(2C) = \frac{w}{2}(L-x)^2$

By comparing coefficients

$EI(2C) = \frac{w}{2}L^2$

$C = \frac{\frac{w}{2}L^2}{2EI}$

$C = \frac{wL^2}{4EI}$

P.I $\Rightarrow \left(\frac{wL^2}{4EI}\right)x^2$

G.S $\Rightarrow$ C.F + P.I

$y = A + Bx + \left(\frac{wL^2}{4EI}\right)x^2$

$\frac{dy}{dx} = B + 2x\left(\frac{wL^2}{4EI}\right)$

at $x = 0,\ y = 0$

$0 = A + B(0) + \left(\frac{wL^2}{4EI}\right)0$

$0 = A$

$A = 0$

at $x = 0,\ dy/dx = 0$

$0 = B + 2(0)\left[\frac{wL^2}{4EI}\right]$

$0 = B$

$B = 0$

$y = 0 + 0x + \left(\frac{wL^2}{4EI}\right)x^2$

$y = \left(\frac{wL^2}{4EI}\right)x^2$

~~at $ab = x$~~ at $x = L$

$y = \left(\frac{wL^2}{4EI}\right)L^2$

$y = \frac{wL^4}{4EI}$