OHUABUNWA REJOICE
15/ENG03/027
CIVIL ENGINEERING
ENG 281

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

let $8 = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 + m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$(m-2)(m+1) = 0$

$m_1 = 2 \quad m_2 = 1$

$y = Ae^{2x} + Be^{-x}$

P.I : $y = c$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4$

P.I $= -4$

G.S = C.F + P.I

$= Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

let $10e^{3x} = 0$

$\frac{d^2y}{dx} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2 \quad m_1 = 2, \quad m_2 = -2$

$y = Ae^{2x} + Be^{-2x}$

P.I : $y = ce^{3x}$

$\frac{dy}{dx} = 3ce^{3x}$

$\frac{d^2y}{dx} = 9ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$

$9ce^{3x} - 4ce^{3x} = 10e^{3x}$

divide through by e^{3x}

$9c - 4c = 10$

$5c = 10$

$c = 2$

P.I = 2

G.S = C.F + P.I

G.S = $Ae^{2x} + Be^{-2x} + 2e^{3x}$

3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Let $C = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1) = 0$ twice

C.F: $y = e^{-x}(A + Bx)$

P.I $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

Divide through by e^{-2x}

$4C - 4C + C = 1$

$C = 1$

G.S = C.F + P.I

G.S = $e^{-x}(A + Bx) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

let $5x^2 + x = 0$

$m^2 + 25 = 0$

$m = \pm\sqrt{-25}$

$m = \pm j5$

C.F: $y = A\cos 5x + B\sin 5x$

P.I: $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$ --- (i)

$25D = 1$ --- (ii)

$2C + 25E = 0$ --- (iii)

from eqn (ii)

$D = \frac{1}{25}$

from eqn (iii)

$2\left(\frac{1}{5}\right) + 25E = 0$

$\frac{2}{5} = -25E$

$\frac{0.4}{-25} = E$

$E = \frac{-2}{125}$

P.I: $y = Cx^2 + Dx + E$

$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

G.S = C.F + P.I

$= A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Let $4\sin x = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$m - 1 = 0$ twice

$m = 1$ twice

C.F: $y = e^x(A + Bx)$

$f(D) = 4\sin x$

P.I: $y = (C\cos x + D\sin x)$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) +$

$C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x +$

$C\cos x + D\sin x = 4\sin x$

$+2C\sin x - 2D\sin x = 4\sin x$

$2C = 4$ ..... (i)

$-2D = 0$ ..... (ii)

$D=0$

$c=\frac{2}{2}=2$

P.I : $y=(\cos x+D\sin x$

$=2\cos x+D\sin x$

$=2\cos x$

G.S $=C.F+P.I$

$=e^{x}(A+Bx)+2\cos x$

6) $\frac{d^2y}{dx^2}+\frac{4dy}{dx}+5y=2e^{-2x}$

given that at $x=0$, $y=1$

and $\frac{dy}{dx}=-2$

let $2e^{-2x}=0$

$m^2+4m+5=0$

$a=1 \quad b=4 \quad c=5$

$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$m=\frac{-4\pm\sqrt{4^2-4\times1\times5}}{2\times1}$

$m=\frac{-4\pm\sqrt{-4}}{2\times1}$

$m=\frac{-4\pm j2}{2\times1}$

$m=-2\pm j \quad \alpha=-2 \quad \beta=1$

C.F : $y=e^{-2x}(A\cos x+B\sin x)$

P.I : $y=ce^{-2x}$

$\frac{dy}{dx}=-2ce^{-2x}$

$\frac{d^2y}{dx^2}=4ce^{-2x}$

$\frac{d^2y}{dx^2}+\frac{4dy}{dx}+5y=2e^{-2x}$

$4ce^{-2x}+4(-2ce^{-2x})+5(ce^{-2x})=2e^{-2x}$

$4ce^{-2x}-8ce^{-2x}+5ce^{-2x}=2e^{-2x}$

divide through by e^{-2x}

$4c-8c+5c=2$

$c=2$

P.I $y=2e^{-2x}$

G.S $=C.F+P.I$

$y=e^{-2x}(A\cos x+B\sin x)+2e^{-2x}$

when $x=0 \quad y=1$

$1=e^{-2(0)}(A\cos(0)+B\sin(0))+2e^{-2(0)}$

$1=1(A+B(0))+2(1)$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$u = e^{-2x}$

$\frac{du}{dx} = -2e^{-2x}$

$v = A\cos x + B\sin x$

$\frac{du}{dx} = -A\sin x + B\cos x$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) +$

$(A\cos x + B\sin x)(-2e^{-2x})$

$-4e^{-2x}$

when $x = 0$ $\frac{dy}{dx} = -2$

$-2 = e^{-2(0)}(2(-1)\sin(0) +$

$B(\cos(0)) + (-1\cos(0) + B\sin(0))(-2) - 4$

$-2 = 1(0 + 2B) + (-1 + 0)(-2) - 4$

$-2 = 2B + 2 - 4$

$-2 = 2B - 2$

$2B = 0$

$B = 0$

$A = -1 \quad B = 0$

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y = e^{-2x}(-1\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

7) $3\frac{d^2y}{dx^3} - 2\frac{dy}{dx} - y = 2x - 3$

let $2x - 3 = 0$

$3m^2 - 2m - 1 = 0$

$3m^3 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(m-1)(3m+1) = 0$

$m_1 = 1 \quad 3m_2 = -1$

$m = -\frac{1}{3}$

6. C.F: $y = Ae^{x} + Be^{-x/3}$

$F(x) = 2x - 3$

$y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$3(2C) - 2(2Cx + D) - (Cx^2 + Dx + E) = 2x - 3$

$6C - 4Cx - 2D - Cx^2 - Dx - E = 2x - 3$

$-Cx^2 = 0$

$C = 0$

$-4C - D = 2$

$-4(0) - D = 2$

$-D = 2$

$D = -2$

$6C - E - 2D = -3$

$6(0) - E - 2(2) = -3$

$-E + 4 = -3$

$-E = -7$

$E = 7$

$y = 0x^2 - 2x + 7$

$y = -2x + 7$

$y = Ae^x + Be^{-x/3} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Let $8e^{4x} = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$(m-4)(m-2) = 0$

$m_2 = -4 \quad m_1 = 2$

$C.F = Ae^{2x} + Be^{4x}$

$y = Ce^{4x}$

$\frac{dy}{dx} = 4Ce^{4x}$

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

C is undefined

Therefore $y = Cxe^{4x}$

$u = Cx \quad v = e^{4x}$

$\frac{du}{dx} = C \quad \frac{dv}{dx} = 4e^{4x}$

$\frac{dy}{dx} = Cx(4e^{4x}) + e^{4x}(C)$

$= 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$= 16Cxe^{4x} + 8Ce^{4x}$

$y = 16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$

$y = 16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$C = 4$

$y = 4xe^{4x}$

G.S $= Ae^{2x} + Bxe^{4x} + 4xe^{4x}$