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CIVIL ENGINEERING

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

Convert equation into an homogenous equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$(m^2 + m)(-2m - 2) = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$(m+1)(m-2) = 0$$

$$m_1 = -1 \quad m_2 = 2$$

$$y = Ae^{-x} + Be^{2x}$$

$$y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2C = 8$$

$$-2C = 8$$

$$C = \frac{8}{-2} = -4$$

$$C = -4$$

$$CS = Ae^{-x} + Be^{2x} - 4$$

$$2) \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$\frac{d^2y}{dx^2} - 4y = 0$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \sqrt{4}$$

$$m = \pm 2$$

$$y = (C \cosh 2x + D \sinh 2x) \quad \text{C.F.}$$

$$y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

divide equation by e^{3x}

$$5C = 10$$

$$C = \frac{10}{5} = 2$$

$$C = 2e^{3x} \quad \text{P.I.}$$

$$\text{GS} = (C \cosh 2x + D \sinh 2x) + 2e^{3x}$$

$$3) \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + (m+1) = 0$$

$(m+1)$ twice

$$y = e^{-x}(A+Bx) \quad \text{C.F.}$$

$$y = C e^{-2x}$$

$$\frac{dy}{dx} = -2C e^{-2x}$$

$$\frac{d^2y}{dx^2} = 4C e^{-2x}$$

$$4C e^{-2x} + 2(-2C e^{-2x}) + C e^{-2x} = e^{-2x}$$

$$4C e^{-2x} - 4C e^{-2x} + C e^{-2x} = e^{-2x}$$

$$C e^{-2x} = e^{-2x}$$

divide equation by e^{-2x}

$$C = 1$$

$$C = e^{-2x} \quad \text{P.I.}$$

$$\text{G.S} = e^{-x}(A+Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm j5$$

$$y = (\cos 5x + D \sin 5x) \quad \text{C.F.}$$

$$y = (x^2 + Dx + E)$$

$$\frac{dy}{dx} = 2x + D$$

$$\frac{d^2y}{dx^2} = 2$$

$$2 + 25(x^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$25C = 5$$

$$C = \frac{5}{25}$$

$$C = \frac{1}{5}$$

$$25D = 1$$

$$D = \frac{1}{25}$$

$$2C + 25E = 0$$

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$\frac{2}{5} + 25E = 0$$

$$25E = -\frac{2}{5}$$

$$E = -\frac{2}{5} \times \frac{1}{25} = -\frac{2}{125}$$

$$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125} \quad \text{P.I.}$$

$$\text{G.S.} = C \cos 5x + D \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$5) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$(m-1) \text{ twice}$$

$$m = 1$$

$$y = e^x (A + Bx) \text{ P.I}$$

$$y = C \cos x + D \sin x$$

$$\frac{dy}{dx} = -C \sin x + D \cos x$$

$$\frac{d^2y}{dx^2} = -C \cos x - D \sin x$$

$$-C \cos x - D \sin x - 2[-C \sin x + D \cos x] + C \cos x + D \sin x = 4 \sin x$$

$$-C \cos x - D \sin x + 2C \sin x - 2D \cos x + C \cos x + D \sin x = 4 \sin x$$

$$-C \cos x - 2D \cos x + C \cos x - D \sin x + 2C \sin x + D \sin x = 4 \sin x$$

$$C \cos x (-C - 2D + C) + \sin x (-D + 2C + D) = 4 \sin x$$

$$-C - 2D + C = 0 \Rightarrow 2D = 0 \quad D = 0$$

$$-D + 2C + D = 4$$

$$2C = 4 \rightarrow C = \frac{4}{2} = 2$$

$$y = 2 \cos x + D \sin x = 2 \cos x \text{ P.I}$$

$$G.S = e^x (A + Bx) + 2 \cos x$$

$$6) \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{3x}$$

given that $x=2, y=1$ and $\frac{dy}{dx} = -2$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$a = 1, b = 4 \text{ and } c = 5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{-4 \pm \sqrt{-4}}{2}$$

$$m = \frac{-4 \pm j2}{2}$$

$$m = -2 \pm j$$

$$m_1 = -2 + j \text{ or } m_2 = -2 - j$$

$$y = e^{-2x} (C \cos x + D \sin x) \text{ P.I}$$

$$y = C e^{-2x}$$

$$\frac{dy}{dx} = -2C e^{-2x}$$

$$\frac{d^2y}{dx^2} = 4C e^{-2x}$$

$$4C e^{-2x} + 4[-2C e^{-2x}] + 5[C e^{-2x}] = 2e^{-3x}$$

$$4C e^{-2x} - 8C e^{-2x} + 5C e^{-2x} = 2e^{-3x}$$

$$4C - 8C + 5C = 2$$

$$C = 2$$

$$y = 2e^{-3x} \text{ P.I}$$

$$y = e^{-2x} (C \cos x + D \sin x) + 2e^{-3x}$$

at $x=0$ and $y=1$

$$1 = e^{-2(0)} [(\cos(0) + D \sin(0))] + 2e^{-2(0)}$$

$$1 = 1 [C + 0] + 2$$

$$1 = C + 2$$

$$C = -2 + 1$$

$$C = -1$$

$$\frac{dy}{dx} = \left[e^{-2x} (-(\sin x + D \cos x)) \right] + \left[2e^{-2x} (\cos x + D \sin x) \right] - 4e^{-2x}$$

when $\frac{dy}{dx} = -2$ $x=0$

$$-2 = [D] + [-2C] \Rightarrow 4$$

$$-2 = D - 2C - 4$$

$$D - 2C = -2 + 4$$

$$D = -2 + 4 + 2(-1)$$

$$D = -2 + 4 - 2$$

$$D = 0$$

$$y = e^{-2x} (\cos x + D \sin x) + 2e^{-2x}$$

$$y = e^{-2x} (-\cos x + D \sin x) + 2e^{-2x}$$

$$y = e^{-2x} (-\cos x + 2)$$

$$P.S = y = e^{-2x} (2 - \cos x)$$

$$\rightarrow \frac{3d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$a = 3, b = -2 \text{ and } -1$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -1}}{2 \times 3}$$

$$m = \frac{2 \pm \sqrt{4 + 12}}{6}$$

$$m = \frac{2 \pm \sqrt{16}}{6}$$

$$m = \frac{2 \pm 4}{6} \Rightarrow m = \frac{1+2}{3}$$

$$m = \frac{1+2}{3} = \frac{3}{3} = 1 \text{ or } m = \frac{1-2}{3} = \frac{-1}{3}$$

$$m_1 = 1 \text{ or } m_2 = -\frac{1}{3}$$

$$y = Ae^{1/3} + Be^{-1/3}$$

$$y = Cx + D$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2y}{dx^2} = 0$$

$$3(0) - 2C - (Cx + D) = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

$$-C = 2$$

$$C = -2$$

$$-2C - D = -3$$

$$4 - D$$

$$D = 4 + 3 = 7$$

$$G.S = Ae^{1/3} + Be^{-1/3} - 2x + 7$$

$$y = 2x + 7$$

$$2) \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$$

$$m^2 - 6m + 8 = 0$$

$$m^2 - 2m - 4m + 8 = 0$$

$$m(m-2) - 4(m-2) = 0$$

$$(m-2)(m-4) = 0$$

$$m_1 = 2 \quad m_2 = 4$$

$$y = Ae^{2x} + Be^{4x} \quad \text{C.F.}$$

$$y = (xe^{4x})$$

$$\frac{dy}{dx} = 4(xe^{4x}) + (e^{4x})$$

$$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + e^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$$

$$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$$

$$2Ce^{4x} = 8e^{4x}$$

$$2C = 8$$

$$C = \frac{8}{2}$$

$$C = 4$$

$$y = 4xe^{4x}$$

P-I

$$\text{Gr-S} = Ae^{2x} + Be^{4x} + 4xe^{4x}$$