$$= \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$$

$$= \cos\left[\sin^{-1}\sqrt{3}/2\right]$$

$$= \cos 60 = 1/2$$

$$\therefore \lim_{x\to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = 1/2$$

d) $\displaystyle\lim_{x\to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

SOLUTION.

$$\lim_{x\to 4}\left[\frac{2x-8}{2x-5}\right]$$

$$= \frac{2(4)-8}{2(4)-5} = 0/3 = 0.$$

$$\therefore \lim_{x\to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right) = 0.$$

2a) $\displaystyle\frac{2}{2\times 3} + \frac{2}{3\times 4} + \frac{2}{4\times 5} + \frac{2}{5\times 6} + \cdots$

SOLUTION

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{2}{(n+1+1)(n+1+2)} = \frac{2}{(n+2)(n+3)}$$

$$\lim_{n\to\infty}\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \div \frac{2}{(n+1)(n+2)}$$

$$= \frac{\cancel{2}}{\cancel{(n+2)}(n+3)} \times \frac{(n+1)\cancel{(n+2)}}{\cancel{2}}$$

$$= \frac{(n+1)}{(n+3)}$$

$$\lim_{n\to\infty}\frac{n+1}{n+3} = \frac{n/n + 1/n}{n/n + 3/n} = \frac{1+1/n}{1+3/n}$$

$n\to\infty$; $1/n \to 0$; $3/n \to 0$

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$$= \frac{1+0}{1+0} = 1$$

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 1$; $\therefore$ the series is either divergent or convergent.

$$\lim_{n\to\infty} u_n \; \frac{2}{(n+1)(n+2)} = \frac{2}{n^2+3n+2}$$

$$= \frac{2/n^2}{1+3/n+2/n^2} = \frac{0}{1} = 0.$$

$\therefore$ the series is convergent since $\lim_{n\to\infty} u_n = 0$.

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

SOLUTION

$$\sum_{n=1}^{\infty} 2/n^p$$

$P = 2$

Since $P > 1$; $\therefore$ the series converges.

c) $u_n = \frac{1+2n^2}{1+n^2}$

SOLUTION

$\lim_{n\to\infty} u_n$

$$\lim_{n\to\infty} \frac{1+2n^2}{1+n^2} = \frac{1/n^2+2}{1/n^2+1}$$

$n\to\infty$; $1/n^2 \to 0$

$$= \frac{0+2}{0+1} = 2.$$

Since $\lim_{n\to\infty} u_n \neq 0$, $\therefore$ the series is divergent.

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3.) $\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

SOLUTION

$U_n = \frac{x^n}{(2n+1)^3}$

$U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+2+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$

$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \div \frac{x^n}{(2n+1)^3}$

$= \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x(2n+1)^3}{(2n+3)^3}$

$\lim_{n\to\infty} \frac{x(2n+1)^3}{(2n+3)^3}$

$\lim_{n\to\infty} \frac{x(8n^3+12n^2+6n+1)}{(8n^3+36n^2+54n+27)}$

$\lim_{n\to\infty} \frac{8xn^3+12xn^2+6xn+x}{8n^3+36n^2+54n+27}$

$= \frac{8x + 12x/n + 6x/n^2 + x/n^3}{8 + 36/n + 54/n^2 + 27/n^3}$

$n\to\infty$; $12x/n \to 0$, $6x/n^2 \to 0$, $x/n^3 \to 0$, $36/n \to 0$, $54/n^2 \to 0$, $27/n^3 \to 0$

$= \frac{8x+0+0+0}{8+0+0+0} = x$

$\lim_{n\to\infty} \frac{x(2n+1)^3}{(2n+3)^3} = x.$

$-1 \leq x \leq 1$, $\therefore$ the series is convergent