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ENG 251
Engineering Maths I

The parametric equations of a curve are as given in Equation 1 & 2

$$x = \cos t + t\sin t$$

$$y = \sin t - t\cos t$$

in term of t determine

I. an expression for the radius of curvature (R), &

II. Expression for the coordinates (h, k) of the centre of Curvature.

Soln

I. $x = \cos t + [t\sin t]$

$x = [t\sin t]$. Let $V = \sin t$, $u = t$

Using product rule

$\frac{du}{dt} = 1$, $u = t$

$V = \sin t$ $\frac{dv}{dt} = \cos t$

$= u\frac{dv}{dt} + v\frac{du}{dt}$

$= t(\cos t) + \sin t(1)$

$= t\cos t + \sin t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$y = \sin t - [t\cos t]$

$y = t\cos t$ Let $u = t$, $V = \cos t$

Using product rule

$u = t$, $\frac{du}{dt} = 1$

$V = \cos t$ $\frac{dv}{dt} = -\sin t$

$= u\frac{du}{dt} + V\frac{dv}{dt}$

$= -t[-\sin t] + \cos t(1)$

$= t\sin t + t\cos t$

$\frac{dx}{dt} = \cos t + t\sin t - \cos t$

$\frac{dy}{dt} = t\sin t$

$$\frac{dy}{dx} = \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$$

* Let $U = \sin t$
  & $V = \cos t$

$$\frac{du}{dt} = \cos t$$

$$\frac{dv}{dt} = -\sin t$$

$$\frac{d^2y}{dx^2} = \frac{\cos t[\cos t] - \sin t[-\sin t)}{(\cos t)^2} \times \frac{dt}{dx}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$$

From trig. identity we have that $\sin^2\theta + \cos^2\theta = 1$

$$\frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

$$R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$= \left[\frac{1}{1} + \frac{\sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \left[\frac{1}{\cos^2 t}\right]^{3/2} \times t\cos^3 t$$

$$= \frac{t\cancel{\cos^3 t}}{\cancel{\cos^3 t}}$$

$$R = t_{//}$$

Expressions for the radius of Curvature R is $t_{//}$

I [h, k]

We call that

$$h = x_1 - R\sin\theta \quad \text{——} (1)$$

$$k = y_1 + R\cos\theta \quad \text{——} (2)$$

$$R = t \quad , \quad \theta = t$$

$$x_1 = \cos t + t\sin t$$

$$y_1 = \sin t - t\cos t$$

Substituting θ, y_1, x_1 & R in equation (1) & (2)

$$h = \cos t + \cancel{t\sin t} - \cancel{t\sin t}$$

$$h = \cos t_{//}$$

$$k = \sin t - \cancel{t\cos t} + \cancel{t\cos t}$$

$$k = \sin t_{//}$$

The expressions for the co-ordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$