SMOOTH SIMON

15/ENG03/030

CIVIL ENG.

Assignment 1

① $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$m - 2 = 0$

$m + 1 = 0$

$m_1 = 2$

$m_2 = -1$

$\therefore$ C.F : $y = Ae^{2x} + Be^{-x}$

$y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx} = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8.$

$0 - 0 - 2C = 8$

P.I : $C = -4$

$y = -4$

$\therefore$ general equation = C.F + P.I

$= Ae^{2x} + Be^{-x} - 4$

② $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4}$

$m = \pm 2$

C.F : $y = C\cosh 2x + D\sinh 2x$

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$C = 5.$

P.I : $y = 5e^{3x}$

General equation : $y = C.F + P.I$

$= C\cosh 2x + D\sinh 2x + 5e^{3x}.$

(3) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + (m+1) = 0$

$(m+1)^2 = 0$

$m = -1$ (force)

$\therefore C.F : y = e^{-x}(A + Bx)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

P.I : $y = e^{-2x}$

General equation = $y = C.F + P.I$

$= e^{-x}(A + Bx) + e^{-2x}.$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m^2 = \pm -25$

$m = \pm\sqrt{25}$

$m = \pm j\sqrt{25}$

$m = \pm j5$

$CF = y = (C\cos 5x + D\sin 5x)$

$y = (Cx^2 + Bx + 8)$

$\frac{dy}{dx} = 2Cx + D + 0 = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C + 0 + 0 = 2C$

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + 8) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25t + 25Cx^2 + 25Dx = 5x^2 + x$

$2C + 25t = 0$ —— ①

$25Cx^2 = 5x^2$ —— ②

$25Dx = x$ —— ③

From ①

$25Cx^2 = 5x^2$

$C = \frac{1}{5}$.

$25Dx = x$

$D = \frac{1}{25}$.

$P_0 + C = \frac{1}{5}$ in ①

$2(\frac{1}{5}) + 25t = 0$

$25t = -\frac{2}{5}$

$E = \frac{-2}{125}$

$P.I = y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

General equation = $y = C.f + P.I$

$= C\cos 5x + D\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$= C\cos 5x + D\sin 5x + \frac{1}{125}(25x^2 + 5x - 2)$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)^2 = 0$

$m = 1$ (form)

C.F : $y = e^x(A + Bx)$

$y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\sin x$

$\frac{d^2y}{dx^2} = -C\cos x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$-(C\cos x - D\sin x + 2(C\sin x + D\sin x) + C\cos x + D\sin x = 4\sin x$

$2C\sin x + 2D\cos x = 4\sin x$

$2\cos x = 4\sin x$

$C = 2$

$2D\cos x = 0$

$D = 0.$

P.I : $y = 2\cos x$

General equation : $y = C.F + P.I$

$= e^x(A + Bx) + 2\cos x$

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CIVIL ENG.

⑥ $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2^{-2x}$

when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$= \frac{-4 \pm j\sqrt{4}}{2}$

$= \frac{-4 \pm j2}{2}$

$m = \frac{-4}{2} \pm j\frac{2}{2}$

$m = -2 \pm j$

$m = \alpha \pm j\beta$

$\therefore C.F = y = e^{-2x}(A\cos x + B\sin x)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

⑦ $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$.

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$.

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$\therefore P.I = y = 2e^{-2x}$

General equation : $y = C.F + P.I$

$y = 2^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$I = e^{-2x} A + 2$

$A = 1 - 2$

$A = -1$

when $\frac{dy}{dx} = -2$

$= e^{-2x} A\cos x + e^{-2x} B\sin x + 2e^{-0x}$

$\frac{dy}{dx} = -2Ae^{-2x}\sin x - 2Be^{-2x}\cos x - 4e^{-2x}$

$-2 = -2Ae^{-2x}\sin 0 + 2Be^{-2x0}\cos 0 - 4e^{-2x0}$

$-2 = -2B - 4$

$-2B = -2 + 4$

$-2B = 2$

$B = -1$

Particular equation : $y = e^{-2x}(-\cos x - \sin x) + 2e^{-2x}$

$= -e^{2x}(\cos x + \sin x) + 2e^{-2x}$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m - m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$m_1 = \frac{-1}{3}$

$m_2 = 1$

C.f : $y = Ae^{\frac{-1}{3}x} + Be^{x}$

$y = \text{Ae}^{0}(x \cdot 1)$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3\frac{d^2y}{dx^2} = 0$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3(0) - 2(C) - Cx + D = 2x - 3$

$-2C - (x - 1)D = 2x - 3$

$-Cx = 2x$

$C = -2$

$-2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$-D = -3 - 4$

$-D = -7$

$D = 7$

$P.I = y = -2x + 7$

General equation: $y = C.F + P.I$

$= Ae^{x} + Be^{-0.33x} - 2x + 7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{6x}$

$m^2 - 6m + 8 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{6 \pm \sqrt{36 - 32}}{2}$

$= \frac{6 \pm \sqrt{4}}{2}$

$= \frac{6 \pm 2}{2}$

$m_1 = \frac{6 + 2}{2} = \frac{8}{2} = 4$

$m_2 = \frac{6 - 2}{2} = \frac{4}{2} = 2$

C.F : $y = Ae^{4x} + Be^{2x}$

$y = Cxe^{4x}$ [because $f(x) = 8e^{4x}$ and C.F: $Ae^{4x} + Be^{2x}$]

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$\frac{d^2y}{dx^2} - \frac{6dy}{dx} + 8y = 8e^{4x}$.

$16Cxe^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$.

$16Cxe^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$.

$4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$Ce^{4x}(2) = 8e^{4x}$

P.I : $y = 4xe^{4x}$

General equation : $y = C.F + P.I$

$= Ae^{4x} + Be^{2x} + 4xe^{4x}$.