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Question

The parametric equations of a curve are as given in Equation (1) and (2).

$$x = \cos t + t\sin t \quad \text{—— (1)}$$
$$y = \sin t - t\cos t \quad \text{—— (2)}$$

In terms of t, determine

(i) an expression for the radius of curvature (R), and.
(ii) Expressions for the coordinates (h, k) of the centre of curvature

Solution

$y = \sin t - t\cos t.$
$x = \cos t + t\sin t.$

$$\frac{dy}{dt} = \cos t - (-t\sin t + \cos t)$$
$$= \cos t + t\sin t - \cos t$$
$$= \cos t - \cos t + t\sin t$$
$$\frac{dy}{dt} = \underline{t\sin t}.$$

$$\frac{dx}{dt} = -\sin t + (t\cos t + \sin t)$$
$$= -\sin t + t\cos t + \sin t$$
$$= t\cos t - \sin t + \sin t.$$
$$\frac{dx}{dt} = \underline{t\cos t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{t\sin t}{t\cos t} = \underline{\tan t}$$

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t\cos t} = \frac{1}{t} \times \frac{1}{\cos^3 t} = t^{-1}\sec^3 t$$

$$\frac{d^2y}{dx^2} = \underline{t^{-1}\sec^3 t}$$

$$R = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left(1+(\tan t)^2\right)^{3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(1+\tan^2 t)^{3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(\sec t)^{2\times 3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(\sec t)^3}{t^{-1}(\sec t)^3} \qquad = \frac{1}{1} \div \frac{1}{t}$$

$$R = t$$

$$\therefore R = \underline{t \text{ units}}$$

Centre of Curvature

$$x_1 = h + R\sin\theta$$
$$h = x_1 - R\sin\theta$$
$$k = y_1 + R\cos\theta$$

$$\theta = \tan^{-1}\left\{\frac{dy}{dx}\right\}$$
$$\theta = \tan^{-1}(\tan t)$$
$$\theta = t$$

$$x_1 = \cos t + t\sin t$$
$$h = \cos t + t\sin t - (t)\sin t$$
$$h = \cos t + t\sin t - t\sin t$$
$$h = \underline{\cos t}$$

$k = y_1 + R\cos\theta$

$y_1 = \sin t - t\cos t$

$k = \sin t - t\cos t + (t)\cos t$

$k = \sin t - t\cos t + t\cos t.$

$k = \sin t.$

Centre of Curvature $= (\overset{h}{\cos t}, \overset{k}{\sin t})$