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Electrical and Electronics Engineering.

Question 1

a) $$\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$$

Solution

l'hopitals rule

$$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Product rule.

$p = (x^2 - \pi/4)(\sin(\cos x))$;

$y = \sin(\cos x)$

function of a function.

$w = \cos x$; $\frac{dw}{dx} = -\sin x$

$y = \sin w$; $\frac{dy}{dw} = \cos w.$

$$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dx}$$

$$= \cos w(-\sin x)$$

$$= -\sin x(\cos w)$$

$$= -\sin x(\cos(\cos x))$$

$$\frac{dy}{dx} = -\sin x(\cos(\cos x))$$

$$\frac{dy}{dx} = \cancel{-\cos \sin x \cos x} = -\underline{\sin x(\cos(\cos x))}$$

$$p = 2x(\sin(\cos x)) + (x^2 - \pi/4)(-\cos \sin x \cos x)$$

$$\lim_{x \to \pi/2} \left(\frac{(x^2 - \pi/4)(-\cos \overset{\cos x}{\cancel{\sin x}} (\overset{\sin x}{\cancel{\cos x}}) + 2x(\sin \cos x))}{1} \right)$$

$$\lim_{n\to\infty} x\left(\frac{8n^3/n^3 + 12n^2/n^3 + 6n/n^3 + 1/n^3}{8n^3/n^3 + 36n^2/n^3 + 54n/n^3 + 27/n^3}\right)$$

$$\lim_{n\to\infty} x\left(\frac{8 + 12/n + 6/n^2 + 1/n^3}{8 + 36n + 54/n^2 + 27/n^3}\right)$$

$$= x\left(8/8\right) = x \times 1$$

$$= \underline{x}$$

For absolute convergence

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| \geq 1$$

$$\underline{\underline{x \geq 1}}$$

Evaluate using L'Hopital's Rule

4) $\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

$$f'(x) = \frac{\cos x + \sin x}{3x^2}$$

$$f''(x) = \frac{-\sin x + \cos x}{6x}$$

$$f'''(x) = \frac{-\cos x - \sin x}{6}$$

when $x \to 0$

$$= \frac{-\cos(0) - \sin(0)}{6}$$

$$= \frac{-1 - 0}{6}$$

$$= -1/6$$

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c) $U_n = \frac{1+2n^2}{1+n^2}$

$$\Rightarrow \frac{1+2(n+1)^2}{1+(n+1)^2}$$

$n \to (n+1)$

~~n~~

$$\Rightarrow \frac{1+2n^2+4n+2}{1+n^2+2n+1} \times \frac{1+n^2}{1+2n^2}$$

$$\Rightarrow \frac{(2n^2+4n+3)(1+n^2)}{(n^2+2n+2)\times(1\times 2n^2)}$$

$$\Rightarrow \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^2}$$

$$\Rightarrow \frac{5n^2+2n^4+4n^3+4n+3}{5n^2+2n^4+4n^3+2n+2}$$

$$\lim_{n\to\infty}\left\{\frac{5/n^2+2+4/n+4/n^3+3/n^4}{5/n^2+2+4/n+2/n^3+2/n^4}\right\}$$

$$= 2/2 = 1 \quad \text{(inconclusive)}$$

(3) Find the range of values of x for which the series below is absolutely convergent:

$$\frac{x}{27}+\frac{x^2}{125}+\dots+\frac{x^n}{(2n+1)^3}$$

$$U_n = \frac{x^n}{(2n+1)^3}, \quad U_{n+1} = \frac{x^{n+1}}{(2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+2+1)^3}$$

$$= \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{\cancel{x^n}+x}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}} = \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\lim_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right| = \lim_{n\to\infty} \cdot \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

2) b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$ Using D'Alemberts ratio.

$U_n = \frac{2}{n^2}$

$U_{n+1} = \frac{2}{(n+1)^2}$

$\lim_{n \to \infty} \frac{2}{n^2+2n+1} \cdot \frac{n^2}{2} = \frac{n^2}{n^2+2n+1} = 1$ which is inconclusive

Testing by comparing with standard P-series

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots \frac{1}{n^p}$

$\frac{2}{1^2} > \frac{1}{1^2}$

$\frac{2}{2^2} > \frac{1}{2^2}$

$\frac{2}{3^2} > \frac{1}{3^2}$

$\frac{2}{4^2} > \frac{1}{4^2}$

$\frac{2}{5^2} > \frac{1}{5^2}$

Hence, the series is divergent.

Determine whether each of the following series is convergent

a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} \cdots$

$U_n = \frac{2}{(n+1)(n+2)}$ Using D'Alembore Ratio Test

$U_{n+1} = \frac{2}{(n+2)(n+3)}$

$\lim_{n\to\infty}\left[\frac{U_{n+1}}{U_n}\right] = \frac{\cancel{2}^{1}}{(\cancel{n+2})(n+3)} \times \frac{(n+1)(\cancel{n+2})}{\cancel{2}} = \frac{(n+1)}{(n+3)}$

$\lim_{n\to\infty} = \frac{n/n + 1/n}{n/n + 3/n} = \frac{1}{1} = 1$ which is inconclusive

so Testing by comparing with standard P-series.

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots \frac{1}{n^p}$

$\frac{2}{2\times3} < \frac{1}{1^2}$

$\frac{2}{3\times4} < \frac{1}{2^2}$

$\frac{2}{4\times5} < \frac{1}{3^2}$

$\frac{2}{5\times6} > \frac{1}{4^2}$ Hence the series is inconclusive.

$= \cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= \cos(\sin^{-1} 0.866)$

$= \cos(60)$

$= \frac{1}{2}$ or 0.5

$\therefore$ The limit of the function is $\frac{1}{2}$ or 0.5

d) $\lim_{x \to 4} \left(\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right) = \frac{(4)^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{0}{0} = 0$

To confirm

Differentiation $\frac{dy}{dx}$

$\lim_{x \to 4} \left(\frac{2x - 8}{2x - 5}\right)$

$\lim_{x \to 4} \left(\frac{2x - 8}{2x - 5}\right)$

$= \frac{2(4) - 8}{2(4) - 5}$

$= \frac{8 - 8}{8 - 5} = \frac{0}{3} = 0$

$\therefore$ The limit of the function is 0

(1) $$\lim_{x \to \pi/2} \ln\left(\exp\left(\frac{3x^2+2x-1}{x+1}\right)\right)$$

Since $\exp = \frac{1}{\ln}$

$$\lim_{x \to \pi/2} \cancel{\ln}\left(\frac{1}{\ln}\left(\frac{3x^2+2x-1}{x+1}\right)\right)$$

$$\lim_{x \to \pi/2} \left(\frac{3x^2+2x-1}{x+1}\right)$$

$$\lim_{x \to \pi/2} \left(\frac{(x-1/3)\cancel{(x+1)}}{\cancel{(x+1)}}\right)$$

$$\therefore \lim_{x \to \pi/2} (x - 1/3)$$

$$= \frac{\pi}{2} - \frac{1}{3} = \frac{3\pi-2}{6}$$

$$= \frac{3\pi-2}{6} = \frac{3\pi}{6} - \frac{2}{6}$$

$$= \frac{3\pi}{6} - \frac{2}{6}$$

$$= \frac{3\pi-2}{6}$$

$\therefore$ The limit of the function $\frac{3\pi-2}{6}$

(3) $$\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\left(\frac{x-2}{x-\sqrt{3}}\right)\right)$$

Substitute directly.

$$\cos\left(\sin^{-1}\left(\frac{\cancel{2}+\sqrt{3}-\cancel{2}}{2+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}}\right)\right)$$

$$= \frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right)\left(-\sin x(\cos\cos x)\right) + 2x\sin(\cos x)}{1}$$

$$= \frac{\left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)\left(-\sin 90^\circ\left(\cos(\overset{0}{\cos 90^\circ})\right) + 2x\sin(\cos 90^\circ)\right)}{1}$$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)\left(-\sin 90^\circ(\cos 0) + 2x\sin 0\right)$$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)\left(-1 \times 1 + 0\right)$$

$$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right)\left(-1 + 0\right)$$

$$= \frac{\pi^2}{4} - \frac{\pi}{4} \times -1$$

$$= -\left(\frac{\pi^2}{4} - \pi/4\right)$$

$$= -\frac{\pi^2}{4} + \pi/4.$$

$\therefore$ The limit of the fenction is $-\frac{\pi^2}{4} + \frac{\pi}{4}$.