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15/ENG07/008
CHEMICAL ENGINEERING
ENG 281 :- ENGINEERING MATHEMATICS III
ASSIGNMENT 1

Solve the following:

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Step 1 :- let R.H.S = 0 and solve to obtain our complementary function

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$m_1 = 1 \qquad m_2 = -1.$$

hence the complementary function; $y = Ae^{x} + Be^{-x}$

To obtain our particular integral

let $y = c$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

Substituting y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the original Equation.

$$0 - 0 - 2(c) = 8$$

$$-2c = 8$$

$$c = -4.$$

Hence P. integral's $y = -4$.

hence general solution = Complementary function + particular integral.

The general solution is given as;

$$y = Ae^{x} + Be^{-x} - 4$$

$$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

Let R.H.S = 0.

$$\frac{d^2y}{dx^2} - 4y = 0$$

$$m^2 - 2^2 = 0$$

$$m^2 = 2^2$$

$$m = \pm 2.$$

$\therefore$ Complementary function; $y = C.\cosh 2x + D\sinh 2x$

To obtain our particular integral

Let $y = Ce^{3x}$

$$\frac{dy}{dx} = 3Ce^{5x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{5x}.$$

Substituting all of the above in the original Question.

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}.$$

$$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$$

$$\cancel{e^{3x}}(9C - 4C) = 10\cancel{e^{3x}}$$

$$5C = 10$$

$$C = 2.$$

Hence our particular integral can be obtained as;

$$y = 2e^{3x}.$$

$\therefore$

The general solution; P.I & C.F

Hence $y = C.\cosh 2x + D\sinh 2x + 2$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

let R.H.S = 0

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

$$m^2 + 2m + 4 = 0$$

$$m = -1 \pm \sqrt{3}i$$

$$m_1 = -1 + \sqrt{3}j \qquad m_2 = -1 - \sqrt{3}j$$

$\therefore$ C.F;

$$y = e^{\alpha x}(\cos\beta x \cdot C + D\sin\beta x)$$

where $\alpha = -1 \quad \beta = \sqrt{3}$

$$y = e^{-x}(\cos\sqrt{3}x \cdot C + D\sin\sqrt{3}x).$$

To obtain our particular integral:

let $y = \cancel{Ce^{-2x}} \quad Cx\,e^{-2x}$ ~~since e^-2x also appears in the~~

$$\frac{dy}{dx} = \cancel{-2Ce^{-2x}} \qquad -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = \cancel{4Ce^{-2x}} \qquad 4Ce^{-2x}.$$

~~Substituting the above in the original Equation~~ subsing the above in the original Eq.

$\cancel{4Ce^{-2x} + 2(-2Ce}$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

hence

our P.I; $y = 1 \cdot e^{-2x} = e^{-2x}$

To obtain the general solution.

G.S; P.I & C.F

$$y = e^{-x}(\cos\sqrt{3}x \cdot C + D\sin\sqrt{3}x) + e^{-2x}$$

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

Let R.H.S $= 0$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$$m^2 + 25 = 0$$

$$m^2 + 5^2 = 0$$

$$m = \pm\sqrt{-5}$$

$$m = \pm j\sqrt{5}$$

hence

C.F $\Rightarrow$ $y = C\cos\sqrt{5}x + D\sin\sqrt{5}x$

To obtain our particular integral

let $y = Ax^2 + Bx + C + Dx + E$

$$\frac{dy}{dx} = 2Ax + B + D$$

$$\frac{d^2y}{dx^2} = 2A$$

Sub all of the above in the original Eqn

$$2A + 25(Ax^2 + Bx + C + Dx + E) = 5x^2 + x$$

$$2A + 25Ax^2 + 25Bx + 25C + 25Dx + 25E = 5x^2 + x$$

$$2A + 25C + 25E + 25Bx + 25Dx + 25Ax^2 = 5x^2 + x$$

$$2A + 25C + 25E + (25B + 25D)x + 25Ax^2 = 5x^2 + x$$

Comparing co-efficient.

$$2A + 25C + 25E = 0 \quad \ldots (i)$$

$$25B + 25D = 1 \quad \ldots (ii)$$

$$25A = 5 \quad \ldots (iii)$$

From Eqn (iii)

$$A = \frac{5}{25} = \frac{1}{5}$$

From Eqn (ii) $\quad 25B + 25D = 1$

$$B + D = \frac{1}{25}$$

$$2\left(\frac{1}{5}\right) + 25C + 25E = 0$$

$$25(C+E) = -\frac{2}{5}$$

$$C+E = -\frac{2}{125}$$

Subbing all of the above in $y = Ax^2 + Bx + C + Dx + E$

$$y = Ax^2 + Bx + Dx + C + E$$

$$y = Ax^2 + (B+D)x + C + E$$

P.I $\Rightarrow$ $y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

To obtain the General Solution;

General Solution ; P.I + C.F

$$\therefore y = C\cos 5x + D\sin 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Let R.H.S $= 0$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$m = 1$ twice , here $m = m_1 = m_2$

$\therefore$

Complementary function; $y = e^{mx}(A + Bx)$

$$y = e^{x}(A + Bx)$$

To obtain our particular integral

Let $y = C\cos x + D\sin x$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$$

Substituting all of the above in the original equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$-C\cos x - 2D\cos x + C\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$$

$$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$$

$$\cos x(-2D) + \sin x(2C) = 4\sin x$$

Comparing co-efficients:

$2C = 4$ ... (i)

$-2D = 0$ ... (ii)

From Equation (i) $C = 2$.

Eqn (ii) $D = 0$.

Substitute the above in the particular integral equation

$y = C\cos x + D\sin x$

$y = 2\cos x$

To obtain our general solution

$G.S = P.I + C.F$

$G.S: \; y = e^{x}(A + Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$ given that $x = 0$, $y = 1$ and $\frac{dy}{dx} = -2$

Let $R.H.S = 0$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$m^2 + 4m + 5 = 0$

$m = -2 \pm j$

where $\alpha = -2$ $\beta = 1$

Complementary function, $y = e^{-2x}(C\cos x + D\sin x)$

To obtain our particular integral

Let $y = Ce^{-2x}$

$$\frac{d^2y}{dx^2} = C\cdot -2e^{-2x} + Cx \cdot 4e^{-2x} + (\cdot 2ce^{-2x})$$

$$\frac{d^2y}{dx^2} = -2ce^{-2x} + 4Cxe^{-2x} - 2ce^{-2x}.$$

$$\frac{d^2y}{dx^2} = -4Ce^{-2x} + 4Cxe^{-2x}. \quad \ldots (4)$$

~~Sub $\frac{d^2y}{dx^2} = \frac{dy}{dx} = -2$ and $x = 0$ in Eq (4).~~

~~Subs all of the above in the original Quan.~~

$\cancel{\frac{dy}{dx} = Cx \cdot -2e^{-2x}}$

$\cancel{-2 =}$

$\cancel{\frac{d^2y}{dx^2} + 4\frac{dy}{dx}} + 5y = 2e^{-2x}$

$\cancel{-4ce^{-2x} + 4cxe^{-2x} + 4(Cx \cdot -2e^{-2x}}$

Subs $\frac{dy}{dx} = -2$ and $x = 0$ in Eq (x)

$$\frac{dy}{dx} = Cx \cdot -2e^{-2x} + Ce^{-2x}$$

$$\frac{dy}{dx} = C(0) \cdot -2e^{-2(0)} + Ce^{-2(0)}$$

$$-2 = Ce^{0}$$

$$-2 = C$$

$$C = -2$$

Re: Sub $C = -2$ in $y = Cx \cdot e^{-2x}$.

P.I $\quad y = -2x \cdot e^{-2x}$

Hence our particular integral, $y = -2xe^{-2x}$.

To obtain the General solution

$$G.S = P.I + C.F$$

$$G.S,\ y = e^{-2x}(C\cdot\cos 2x + D\sin x) - 2xe^{-2x}$$

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

Let R.H.S $= 0$

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$$

$$3m^2 - 2m - 1 = 0$$

$$m_1 = 1 \quad m_2 = -\frac{1}{3}$$

Hence

Complementary function, $y = Ae^{x} + Be^{-\frac{1x}{3}}$.

To obtain the particular integral

PI => $y = Cx + D + E$

$\frac{dy}{dx} = c$

$\frac{d^2y}{dx^2} = 0$

Substitute all of the above in the original Question

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3(0) - 2(C) - Cx - D - E = 2x - 3$

$-2C - Cx - D - E = 2x - 3$

$-2C - D - E - Cx = 2x - 3$

Comparing Co-efficients:

$-C = 2 \quad \ldots (i)$.

$-2C - D - E = -3 \quad \ldots (ii)$

From Eqn (i) $C = -2$

Subst $C = -2$ in Eqn (ii)

$-2(-2) - D - E = -3$

$4 - D - E = -3$

$-D - E = -3 - 4$

$-D - E = -7$

$D + E = 7 \quad \ldots (iv)$

Subst all of the above in the particular integral

PI => $y = Cx + D + E$

$y = -2x + 7$

$\therefore$ General Solution, C.F + P.I

$y = Ae^{x} + Be^{-\frac{1}{3}x} - 2x + 7$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Let R.H.S = 0

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m_1 = 4, \quad m_2 = 2.$

hence C.F, $y = Ae^{4x} + Be^{2x}$.

To obtain our particular integral, since e^{4x} is already in the complementary function, let $y = Cxe^{4x}$

then $\frac{dy}{dx} = C \cdot e^{4x} + Cx \cdot 4e^{4x}$

$\frac{d^2y}{dx^2} = 4Ce^{4x} + C \cdot 4e^{4x} + Cx \cdot 16e^{4x}$

Substitute all of the above in the original question

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$4Ce^{4x} + C \cdot 4e^{4x} + Cx \cdot 16e^{4x} - 6(C \cdot e^{4x} + Cx \cdot 4e^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$4Ce^{4x} + 4Ce^{4x} + 16Cxe^{4x} - 6Ce^{4x} - 6Cx \cdot 4e^{4x} + 8Cxe^{4x} = 8e^{4x}$

$8Ce^{4x} + 16Cxe^{4x} - 6Ce^{4x} - 24Cxe^{4x} + 8Cxe^{4x} = 8e^{4x}$

$8Ce^{4x} - 6Ce^{4x} + 16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} = 8e^{4x}$

$2Ce^{4x} + 24Cxe^{4x} - 24Cxe^{4x} = 8e^{4x}$

$\frac{2Ce^{4x}}{2e^{4x}} = \frac{8e^{4x}}{2e^{4x}}$

$C = 4$

Subst $C = 4$ in $y = Cxe^{4x}$

Hence P.I $\Rightarrow$ $y = 4xe^{4x}$.

$\therefore$ The General Solution $=$ P.I + C.F

General Solution $y = Ae^{4x} + Be^{2x} + 4xe^{4x}$