ORIOLA MUMUIRAT OMOBOLAJI
16/ENG07/024
PETROLEUM ENGINEERING.
ENG 281. Engineering mathematics I.

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

Recall from diagram : $h = x_1 - LP = x_1 - R\sin\theta$

$k = y_1 - LC = y_1 + R\cos\theta$

In terms of t:

$h = x_1 - R\sin t$

$k = y_1 + R\sin t.$

$$R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

* $\frac{dx}{dt} = t\cos t$

using product rule to differentiate $t\sin t$ $\left[U\frac{dv}{dt} + V\frac{du}{dt}\right]$

$x = \cos t + t\sin t$

Let $u = t\sin t$, $\frac{du}{dx} = t\cos t + \cancel{\cos t}\ \sin t$

$\frac{dx}{dt} = -\cancel{\sin t} + t\cos t + \cancel{\sin t}$

$\frac{dx}{dt} = t\cos t.$

$\frac{dy}{dt} = t\sin t.$

Using product rule to differentiate $-t\cos t$ $\left[U\frac{dv}{dt} + V\frac{du}{dt}\right]$.

Let $V = -t\cos t$, $\frac{dv}{dt} = t\sin t - \cos t.$

$\frac{dy}{dt} = \cancel{\cos t} + t\sin t - \cancel{\cos t}$

$\therefore \frac{dy}{dt} = t\sin t.$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{\cancel{t}\sin t}{\cancel{t}\cos t} = \tan t$

$\frac{d^2y}{dx^2} = \frac{d}{dx}.\frac{dy}{dx}.\frac{dt}{dt} = \frac{d}{dt}.\frac{dt}{dx}.\frac{dy}{dx}$

$\therefore \frac{d^2y}{dx^2} = \frac{1}{t\cos t} \times \sec^2 t$

$\frac{d^2y}{dx^2} = \frac{1}{t\cos t} \times \frac{1}{\cos^2 t} = \frac{1}{t} \times \frac{1}{\cos t} \times \frac{1}{\cos^2 t}$

$\frac{d^2y}{dx^2} = \frac{1}{t} \times \frac{1}{\cos^3 t}$

$\frac{d^2y}{dx^2} = t^{-1}\sec^3 t$

Scanned by CamScanner

$$R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$R = \frac{\left[1 + [\tan t]^2\right]^{3/2}}{t^{-1}\sec^3 t} = \frac{[1 + \tan^2 t]}{t^{-1}\sec^3 t}$$

From: $1 + \tan^2 t = \sec^2 t$

$$R = \frac{\left[(\sec t)^2\right]^{3/2}}{t^{-1}\sec^3 t} = \frac{(\sec t)^3}{t^{-1}(\sec^3 t)}$$

$$R = \frac{\cancel{\sec^3 t} \cdot t}{\cancel{\sec^3 t}} = t$$

$\therefore R = t$

$$\tan\theta = \left\{\frac{dy}{dx}\right\}_p$$

$\tan\theta = \tan t$

$\theta = \tan^{-1}[\tan t]$

$\theta = t$

$h = x_1 - R\sin\theta$

$h = \cos t + t\sin t - [t\sin t]$

$\boxed{\therefore h = \cos t}$

$k = y_1 + R\cos\theta$

$k = \sin t - t\cos t + t\cos t$

$\therefore \boxed{k = \sin t}$

$\therefore$ The coordinates of the centre of curvature $C(h, k)$ is $(\cos t, \sin t)$