Ass 2

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1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

i) Complementary function

$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 0$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1 \cdot 5)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm j\sqrt{4}}{2} = -2 \pm j$

$y = e^{-2\theta}(A\cos\theta + B\sin\theta)$

Partial integration

$y = C\cos\theta + D\sin\theta$

$\frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$\frac{d^2y}{d\theta^2} = -C\cos\theta - D\sin\theta$

$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$\cos\theta(-C + 4D + 5C) + \sin\theta(-D - 4C + 5D) = 6\sin\theta$

$\cos\theta\,(4C + 4D) + \sin\theta\,(4D - 4C) = 6\sin\theta$

Comparing RHS & LHS

$+4C + 4D = 0 \quad -(1)$

$4D - 4C = 6 \quad -(2)$

$\cancel{4D = 4C}$

$\cancel{D = \frac{4C}{4} = C}$

$\cancel{4D - 4C = 6}$

$\cancel{4(C) - 4C = 6}$

$\cancel{0 = 6}$

from (1)

$C + D = 0$

$C = -D$

$4D - 4(-D) = 6$

$4D + 4D = 6$

$8D = 6$

$D = \frac{6}{8} = \frac{3}{4}$

$\therefore C = \frac{-3}{4}$

$P.I.\ y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

General Solution;

$$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$$

At steady state $\theta = \infty$ $\frac{dy}{d\theta} = 0$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(y)$

$U = e^{-2\theta}$

$\frac{du}{d\theta} = -2e^{-2\theta}$

$V = A\cos\theta + B\sin\theta$

$\frac{dv}{d\theta} = -A\sin\theta + B\cos\theta$

$$e^{-2t}(-A\sin t + B\cos t) + (A\cos\theta + B\sin\theta) - 2e^{-2\theta}$$

$$dy/d\theta = e^{-2t}(-A\sin t + B\cos\theta) + (A\cos\theta + B\sin t)(-2e^{-2\theta}) + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$$\theta = \infty \qquad dy/d\theta = 0$$

$$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

divide through by $\cos\theta$

$$0 = \frac{3}{4}\tan\theta + \frac{3}{4}$$

$$-\frac{3}{4} = \frac{3}{4}\tan\theta$$

$$\frac{-3 \times \cancel{4}^{1}}{\cancel{4}_{1}} = 3\tan\theta$$

$$-3 = 3\tan\theta$$

$$-\frac{3}{3} = \tan\theta$$

$$-1 = \tan\theta$$

$$\theta = \tan^{-1}[-1]$$

$$\theta = -45^{\circ}$$

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$L^2 - 2xL + x^2 \quad a^2 + 2ab + b^2$

2) $EI \frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

Complementary fxn

$$EI \frac{d^2y}{dx^2} = 0$$

let $\frac{d^2y}{dx^2} = m^2$

$$EI(m^2) = 0$$

$$m^2 = 0$$

$$m = 0$$

$$y = A\cos 0 + B\sin 0$$

$$y = A$$

Particular fxn

$$y = \frac{w}{2}(L-x)^2$$

$$y = Cx^2 + Dx + E$$

$$\frac{d^2y}{dx^2} \cancel{= 2Cx + D} \; 2C$$

$$EI(2C) = \frac{w}{2}(L-x)^2$$

$$EI(2C) = \frac{wL^2}{2} - \frac{2xLw}{2} + \frac{wx^2}{2}$$

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