Assignment 2

① $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

ⓐ Obtain an expression for y as a function of θ.
ⓑ Neglecting the complementary function of the general solution, with the aid of Microsoft excel, plot the response of the system for $\theta = 0$ to 270°.
ⓒ Using the expression obtained in ⓐ, estimate the value of θ at steady state.

Soln

ⓐ $m^2 + 4m + 5 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm j\sqrt{4}}{2}$$

$$= \frac{-4}{2} \pm j\frac{2}{2}$$

$m = -2 \pm j$

$m = \alpha \pm j\beta$

$y = \cancel{Acos}\, e^{-2\theta}(A\cos\theta + B\sin\theta)$ [C.F]

$y = C\cos\theta + D\sin\theta$

$$\frac{dy}{dx} = -C\sin\theta + D\cos\theta$$

$$\frac{d^2y}{dx^2} = -C\cos\theta - D\sin\theta$$

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$$\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$$

$$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$$

$$-4C\sin\theta + 4D\cos\theta + 4C\cos\theta + 4D\sin\theta = 6\sin\theta$$

$$-4C\sin\theta + 4D\sin\theta = 6\sin\theta$$

$$(-4C + 4D)\cancel{\sin\theta} = 6\cancel{\sin\theta}$$

$$-4C + 4D = 6 \quad \text{—— } (1)$$

$$4D\cos\theta + 4C\cos\theta = 0$$

$$4D + 4C = 0 \quad \text{—— } (2)$$

from (2)

$$4D = -4C$$

$$D = -C$$

Put that in (1)

$$-4C + 4(-C) = 6$$

$$-4C - 4C = 6$$

$$-8C = 6$$

$$C = -\frac{6}{8}$$

$$C = -\frac{3}{4}$$

$$\therefore D = 3/4$$

[P.I] : $y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

$\therefore$ G.S : C.F + P.I

$$y = e^{-2\theta}(A\cos\theta + B\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$$

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$$\frac{dy}{d\theta} = 0 \;, \quad \theta = \infty$$

$$\frac{dy}{d\theta} = -e^{-2\theta} A\sin\theta - 2e^{-2\theta} A\cos\theta + e^{-2\theta} B\cos\theta - 2e^{-2\theta} B\sin\theta + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$$0 = -e^{-2\theta} A\sin\theta - 2e^{-2\theta} B\sin\theta + \frac{3}{4}\sin\theta - 2e^{-2\theta} A\cos\theta + e^{-2\theta} B\cos\theta + \frac{3}{4}\cos\theta$$

at steady state;

$$\frac{dy}{d\theta} = 0 \;, \quad \theta = \infty$$

$$\frac{dy}{d\theta} = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$$

$$0 = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$$

$$\frac{3}{4}\sin\theta = \frac{3}{4}\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1} 1$$

$$= 45^\circ$$

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(2) $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

<u>Soln</u>

$EIm^2 = 0$

$m^2 = 0$

$m = \sqrt{0}$

$m = 0$

C.F: $y = e^{0x}(A+Bx)$

$= A+Bx$

$y = Cx^2 + Dx^3 + Ex^4$

$\frac{dy}{dx} = 2Cx + 3Dx^2 + 4Ex^3$

$\frac{d^2y}{dx^2} = 2C + 6Dx + 12Ex^2$

$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

$EI(2C + 6Dx + 12Ex^2) = \frac{w}{2}(L-x)^2$

$EI2C + EI6Dx + EI12Ex^2 = \frac{w}{2}(L^2 - 2Lx + x^2)$

$EI4C + EI12Dx + EI24Ex^2 = wL^2 - w2Lx + wx^2$

$EI24E\cancel{x^2} = w\cancel{x^2}$

$E = \frac{w}{24EI}$

$EI12D\cancel{x} = -w2L\cancel{x}$

$D = \frac{-2wL}{12EI}$

$EI4C = wL^2$

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$$C = \frac{wl^2}{4EI}$$

$$P.I: y = \frac{wl^2}{4EI}x^2 + \frac{-2wl}{12EI}x^3 + \frac{w}{24EI}x^4$$

$G.S: C.F + P.I$

$$y = A + Bx + \frac{wl^2}{4EI}x^2 - \frac{2wl}{12EI}x^3 + \frac{w}{24EI}x^4$$

$$y = A + Bx + \frac{wl^2}{4EI}x^2 - \frac{wl}{6EI}x^3 + \frac{w}{24EI}x^4$$

at $y = 0,\ x = 0,\ \frac{dy}{dx} = 0$

$$0 = A + 0 + 0 + 0$$

$\therefore\ A = 0$

$$\frac{dy}{dx} = B + 2\frac{wl^2}{4EI}x - 3\frac{wl}{6EI}x^2 + 4\frac{w}{24EI}x^3$$

$$0 = B + 0 + 0 + 0$$

$$B = 0$$

$\therefore$ Particular equation; $y = \frac{wl^2}{4EI}x^2 - \frac{wl}{6EI}x^3 + \frac{w}{24EI}x^4$

when $x = L$

$$y = \frac{wl^4}{4EI} - \frac{wl^4}{6EI} + \frac{wl^4}{24EI}$$

$$y = \frac{6wl^4 - 4wl^4 + wl^4}{24EI}$$

$$y = \frac{1}{24EI}(6wl^4 - 4wl^4 + wl^4)$$

$$= \frac{3wl^4}{24EI} \Rightarrow \frac{wl^4}{8EI}$$

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