

ENGG 351

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COMPUTER ENGINEERING

ISENG102 2016

$$\textcircled{1} \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4$$

$$m^2 - m - 2 = 0 \quad m_1 = 2, m_2 = -1$$

$$y = Ae^{2x} + Be^{-x}$$

$$P_1: y = c$$

$$\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2c = 4$$

$$-2c = 4$$

$$c = \frac{-4}{2}$$

$$c = -2$$

$$P_1: y = -2$$

$$G.S: y = Ae^{2x} + Be^{-x} - 2$$

$$\textcircled{2} \frac{d^2y}{dx^2} - 4y = 10e^{3x}$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm \sqrt{4}$$

$$m = \pm 2j$$

$$y = C \cosh 2x + D \sinh 2x$$

$$P_1: y = Ce^{3x}$$

$$\frac{dy}{dx} = 3Ce^{3x}$$

$$\frac{d^2y}{dx^2} = 9Ce^{3x}$$

$$9Ce^{3x} = 4(Ce^{3x}) = 10e^{3x}$$

$$5Ce^{3x} = 10e^{3x}$$

$$C = \frac{10e^{3x}}{5e^{3x}}$$

$$C = 2$$

$$G.S = C \cosh 2x + D \sinh 2x + 2e^{3x}$$

$$\textcircled{3} \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$m + 2m + 1 = 0$$

$$\frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-2}{2} = -1$$

$$m = -1$$

$$y = e^{-x} (A + Bx + C)$$

$$\text{P.I. : } y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4Ce^{-2x} - 2(2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$y = e^{-2x}$$

$$\text{G.S.} = y = e^{-x} (A + Bx) + e^{-2x}$$

$$\textcircled{4} \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm \sqrt{25}$$

$$m = \pm 5j$$

$$y = C \cos 5x + D \sin 5x$$

$$\text{P.I. : } y = Cx^2 + Dx + E$$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$25C = 5 \text{ comparing coefficients } C = 1/5$$

$$25D = 1 \quad ; \quad D = 1/25$$

$$25E + 2C = 0$$

$$25E + 2(1/5) = 0$$

$$25E = -2/5 \quad ; \quad E = -2/125$$

$$y = 1/5 x^2 + 1/25 x - 2/125$$

$$\text{G.S.} = y = C \cos 5x + D \sin 5x + 1/5 x^2 + 1/25 x - 2/125$$

$$\textcircled{5} \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$m^2 - 2m + 1$$

$$2 + \frac{\sqrt{(-2)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{2}{2} = 1$$

$$m = 1$$

$$y = e^x (A + Bx)$$

$$\frac{dy}{dx} = -C\sin x + D\cos x$$

$$\frac{d^2y}{dx^2} = C\cos x - D\sin x$$

$$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$= -2D\cos x + 2C\sin x = 4\sin x$$

comparing coefficients

$$-2D = 0 \quad 2C = 4$$

$$D = 0 \quad C = 2$$

$$D = 0 \quad C = 2$$

$$y = 2\cos x + 0\sin x$$

$$y = 2\cos x$$

$$\text{G.S.} = y = e^x (A + Bx)$$

$$\textcircled{c} \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

$$m^2 + 4m + 5$$

$$y = e^{-2x} (\cos 2x + D \sin 2x)$$

$$P_1: y = Ce^{-2x}$$

$$\frac{dy}{dx} = -2Ce^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$4(Ce^{-2x}) + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$$

$$4C - 8C + 5C = 2$$

$$-C = 2 \Rightarrow C = -2$$

$$C = 2, y = 2e^{-2x}$$

$$\text{G.S. } y = e^{-2x} (\cos 2x + D \sin 2x) + 2e^{-2x}$$

When $x = 0$

$$y = e^{-2(0)} (C(\cos 2(0)) + D \sin 2(0)) + 2e^{-2(0)}$$

$$1 = C + 2 \Rightarrow C = -1$$

$$\frac{dy}{dx} = e^{-2x} (-2D \sin 2x + 2C \cos 2x) + (-2C - 2D)e^{-2x}$$

$$\text{at } x = 0$$

$$0 = -1 - 2D$$

$$\text{G.S. } y = e^{-2x} (\cos 2x - D \sin 2x)$$

$$\textcircled{2} \quad 3 \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

$$3m^2 - 2m - 1 = 0$$

$$2 \pm \frac{\sqrt{(2)^2 - 4 \times 3 \times (-1)}}{2 \times 3} = \frac{2 \pm \sqrt{4 + 12}}{6}$$

$$= \frac{2 \pm \sqrt{16}}{6}$$

$$= \frac{2 + 4}{6} \quad \text{or} \quad \frac{2 - 4}{6}$$

$$= 1 \quad \text{or} \quad -\frac{1}{3}$$

$$m_1 = 1 \quad ; \quad m_2 = -\frac{1}{3}$$

$$y = A e^x + B e^{-\frac{1}{3}x}$$

$$P.I: y = Cx + D$$

$$\frac{dy}{dx} = C$$

$$\frac{d^2 y}{dx^2} = 0$$

$$3(Cx + D) - 2C - (Cx + D) = 2x - 3$$

$$\rightarrow C - Cx - D = 2x - 3$$

comparing coefficients

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 4 + 3 = 7$$

$$y = -2x + 7$$

$$\therefore y = A e^x + B e^{-\frac{1}{3}x} - 2x + 7$$

$$\textcircled{9} \quad \frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 8e^{4x}$$

$$m^2 - 6m + 9 = 0$$

$$2(1) = 4 \times 2 = 2$$

$$y = Ae^{4x} + Be^{2x} = e^{4x}$$

$$\text{P.I. } y = Cxe^{4x}$$

$$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$$

$$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 8Ce^{4x}$$

$$16Cxe^{4x} + 8Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 9(Cxe^{4x}) = 8e^{4x}$$

$$16C + 8C - 24Cx - 6C + 9Cx = 8$$

$$2C = 8$$

$$C = 4$$

$$y = 4xe^{4x}$$

$$\text{Ans: } y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$