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DEPARTMENT: CHEMICAL ENGINEERING

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1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$(m+1)(m-2) = 0$

$m = -1$ or 2

$y = Ae^{-x} + Be^{2x}$

For particular integration; $y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2C = 8$

$C = \frac{8}{-2}$

$C = -4$

G.S; $y = Ae^{-x} + Be^{2x} - 4$

2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = \pm\sqrt{4}$

$m^2 = \pm 2$

$y = A\cosh 2x + B\sinh 2x$

For particular integration; $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$\frac{5C}{5} = \frac{10}{5}$

$C = 2$

$\therefore y = 2e^{3x}$

G.S; $y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

3 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

For particular integration; $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$C = 1$

$\therefore y = e^{-2x}$

G.S; $y = e^{-x}(A + Bx) + e^{-2x}$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$\frac{d^2y}{dx^2} + 25y = 0$

$m^2 + 25 = 0$

$m^2 = \pm\sqrt{-25}$

$m = \pm 5j$

$y = A\cos 5x + B\sin 5x$

For particular integration; $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$2C + 25E = 0 \quad \cdots (i)$

$25D = 1 \quad \cdots (iii)$

$25C = 5 \quad \cdots (iv)$

From equ (iii)

$D = \frac{1}{25}$

From equ (iv)

$C = \frac{5}{25}$

$= \frac{1}{5}$

Substituting C in equ (i)

$2(\frac{1}{5}) + 25E = 0$

$\frac{2}{5} + 25E = 0$

$25E = -\frac{2}{5}$

$E = -\frac{2}{5} \times \frac{1}{25}$

$E = -\frac{2}{125}$

$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

G.S; $y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = A\cos 5x + B\sin 5x + \frac{1}{5}(x^2 + \frac{1}{5}x - \frac{2}{25})$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx)$

For particular integration; $y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$\cos x(-C-2D)+\sin x(-D+2C+D)=4\sin x$

$\frac{-2D}{-2}=\frac{0}{-2}$

$D=0$

$\frac{2C}{2}=\frac{4}{2}$

$C=2$

$y=2\cos x$

(c.s) $y=e^{x}(A+Bx)+2\cos x$

6 $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=2e^{-2x}$

$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=0$

$m^2+4m+5=0$

$a=1, b=4, c=5$

$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-4\pm\sqrt{4^2-4(1)(5)}}{2(1)}$

$=\frac{-4\pm\sqrt{16-20}}{2}$

$=\frac{-4\pm\sqrt{-1}\cdot\sqrt{4}}{2}$

$=\frac{-4\pm 2j}{2}$

$m=-2\pm j$

$y=e^{-2x}(A\cos x+B\sin x)$

For particular integration; $y=Ce^{-2x}$

$\frac{dy}{dx}=-2Ce^{-2x}$

$\frac{d^2y}{dx^2}=4Ce^{-2x}$

$4Ce^{-2x}-8Ce^{-2x}+5Ce^{-2x}=2e^{-2x}$

$Ce^{-2x}=2e^{-2x}$

$C = 2$

$y = 2e^{-2x}$

G.S; $y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x = 0, y = 1$

$1 = e^{-2(0)}(A\cos 0 + B\sin 0) + 2e^{-2(0)}$

$1 = 1(A) + 2$

$1 = A + 2$

$A = 1 - 2$

$A = -1$

at $x = 0, \frac{dy}{dx} = -2$

$\frac{dy}{dx} = -2e^{2x}(A\cos x + B\sin x) + e^{-2x}(-A\sin x + B\cos x) - 4e^{-2x}$

$-2 = -2e^{2(0)}(A\cos 0 + B\sin 0) + e^{-2(0)}(-A\sin 0 + B\cos 0) - 4e^{-2(0)}$

$-2 = -2(A) + B - 4$

$-2 = -2(-1) + B - 4$

$-2 = 2 + B - 4$

$B = -2 - 2 + 4$

$B = 0$

P.S; $y = e^{-2x}(-\cos x) + 2e^{-2x}$

~~$y = -2e^{-2x}$~~ $y = -e^{-2x}\cos x + 2e^{-2x}$

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

$3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(m-1)(3m+1) = 0$

$m = 1$ or $m = -\frac{1}{3}$

$y = Ae^{-\frac{1}{3}x} + Be^{x}$

For particular integration; $y = (x + 1)$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$-2C - Cx - D = 2x + 3$

Comparing coefficients

$-2C - D = -3 \quad \cdots (i)$

$-C = 2 \quad \cdots (ii)$

$C = -2$

Substitute $C = 2$ in eqn (i)

$-2(-2) - D = -3$

$+4 - D = -3$

$+4 + 3 = D$

$D = +7$

$y = -2x + 7$

G.S; $y = Ae^{-\frac{1}{3}x} + Be^{x} - 2x + 7$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{+x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-2)(m-4) = 0$

$m = 2$ or $m = 4$

$y = Ae^{2x} + Be^{4x}$

For particular integration; $y = Cxe^{4x}$

$$\frac{dy}{dx} = C(e^{4x} + 4xe^{4x}) = 4Cxe^{4x} + Ce^{4x}$$

$$\frac{d^2y}{dx^2} = 4C(e^{4x} + 4xe^{4x}) + 4Ce^{4x}$$

$$= 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$$

$$= 16Cxe^{4x} + 8Ce^{4x}$$

$$16Cxe^{4x} + 8Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$2Ce^{4x} = 8e^{4x}$$

$$\frac{2C}{2} = \frac{8}{2}$$

$$C = 4$$

$$y = 4xe^{4x}$$

G.S; $y = Ae^{2x} + Be^{4x} + 4xe^{4x}$