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①

$$\frac{d^2 y}{d\theta^2} + 4 \frac{dy}{d\theta} + 5y = 6 \sin \theta$$

$$m^2 + 4m + 5 = 0$$

$$a = 1, b = 4, c = 5$$

$$-b \pm \sqrt{b^2 - 4ac}$$

$$\frac{-4 \pm \sqrt{4^2 - 4 \times 5 \times 1}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2}$$

$$m = -2 \pm j$$

$$c.f = e^{-2\theta} [C \cos \theta + D \sin \theta]$$

$$P.I \Rightarrow y = 6 \sin \theta$$

$$y = C \sin \theta + D \cos \theta$$

$$\frac{dy}{d\theta} = C \cos \theta - D \sin \theta$$

$$\frac{d^2 y}{d\theta^2} = -C \sin \theta - D \cos \theta$$

$$-C \sin \theta - D \cos \theta + 4[C \cos \theta - D \sin \theta] + 5[C \sin \theta + D \cos \theta] = 6 \sin \theta$$

$$-C \sin \theta - D \cos \theta + 4C \cos \theta - 4D \sin \theta + 5C \sin \theta + 5D \cos \theta = 6 \sin \theta$$

$$4C \sin \theta + 4D \cos \theta + 4C \cos \theta - 4D \sin \theta = 6 \sin \theta$$

$$4C - 4D = 6$$

$$4D + 4C = 0$$

$$4C - 4D = 6$$

$$+ 4C + 4D = 0$$

$$8C = 6$$

$$C = \frac{6}{8} = \frac{3}{4}$$

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Sub $\frac{3}{4}$ for C in equ ①

$$4 \left(\frac{3}{4} \right) - 4D = 6$$

$$3 - 4D = 6$$

$$-4D = 6 - 3$$

$$-4D = 3$$

$$D = -\frac{3}{4}$$

$$P.I = \frac{3}{4} \sin \theta + \left(-\frac{3}{4} \cos \theta \right)$$

$$P.I = \frac{3}{4} \sin \theta - \frac{3}{4} \cos \theta$$

$$C.I.S \rightarrow e^{-2\theta} \left[C \cos \theta + D \sin \theta \right] + \frac{3}{4} \sin \theta - \frac{3}{4} \cos \theta$$

(ii) At steady state

$$y'' = 0$$

$$y = \frac{3}{4} \sin \theta - \frac{3}{4} \cos \theta$$

$$y'' = \frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta$$

$$\frac{3}{4} \cos \theta + \frac{3}{4} \sin \theta = 0$$

$$\frac{3}{4} \cos \theta - \frac{3}{4} \sin \theta$$

$$-\frac{3}{4} = \frac{3}{4} = \frac{\sin \theta}{\cos \theta}$$

$$\cos \theta$$

$$-1 = \tan \theta$$

$$\tan \theta =$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

\therefore At steady state $\theta = -45^\circ$