JASPER VICTORY
16/ENG03/033
CIVIL ENGINEERING
ENG 281

Assignment 2

The parametric eqns of a curve are as given eqns (1) and (2).

$x = \cos t + \sin t$

$y = \sin t - t\cos t$

In terms of t, determine

(i) an expression for the radius of curvature (R) and

(ii) expressions for the coordinates (h, k) of the centre of curvature.

Solution

(i) Radius of curvature $= \dfrac{(1+(dy/dx)^2)^{3/2}}{d^2y/dx^2}$

~~$R = (1+\tan^2 t)^{3/2}$~~

$x = \cos t + \sin t$

$\dfrac{dx}{dt} = -\sin t + \sin t + t\cos t$

$\dfrac{dx}{dt} = t\cos t$

$\dfrac{dt}{dx} = \dfrac{1}{t\cos t}$

$y = \sin t - t\cos t$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$= t\sin t \times \dfrac{1}{\cos t}$

$\dfrac{dy}{dx} = \dfrac{t\sin t}{t\cos t}$

$= \tan t$

$\dfrac{d^2y}{dx^2} = \sec^2 t \times \dfrac{dt}{dx}$

$= \sec^2 t \times \dfrac{1}{t\cos t}$

$= \dfrac{\sec^2 t}{t\cos t}$

$R = \dfrac{(1+\tan^2 t)^{3/2}}{\sec^2 t / t\cos t}$

$R = \dfrac{(\sec^2 t)^{3/2}}{\sec^2 t / t\cos t}$

$R = \sec^3 t \div \dfrac{\sec^2 t}{t\cos t}$

$= \dfrac{\sec^3 t \times t\cos t}{\sec^2 t}$

$R = \sec t \times t\cos t$

Recall $\sec t = \dfrac{1}{\cos t}$

$R = \dfrac{1}{\cos t} \times t\cos t$

$R = t$ //

(ii) $h = x_1 - R\sin t$

$k = y_1 + R\cos t$

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

Coordinate $(\cos t, \sin t)$.