JASPER VICTORY
16/ENG03/033
CIVIL ENGINEERING
ENG 281

Assignment 1

1\. a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right]$

Solution

Let $u = x^2 - \pi/4$

$\frac{du}{dx} = 2x.$

and $v = \sin(\cos x)$

let $t = \cos x$ and $v = \sin t$

$\frac{dt}{dx} = -\sin x$ $\qquad \frac{dv}{dt} = \cos t$

$\frac{dv}{dx} = \frac{dt}{dx} \times \frac{dv}{dt} = -\sin \cos t$

$\frac{dv}{dx} = -\sin(\cos(\cos x))$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= [x^2 - \pi/4] - \cos(\cos x)\sin x + \sin(\cos x)2x$

Denominator

Let $a = x - \pi/2$

$da/dx = 1$

$\therefore \frac{(x^2 - \pi/4)(-\cos(\cos x)\sin x) + \sin(\cos x)2x}{1}$

$\lim_{x \to \pi/2} (\pi^2/2^2 - \pi/4)(-\cos(\cos \pi/2)\sin \pi/2) +$
$\sin(\cos \pi/2) 2 \pi/2$

$= (\pi^2/4 - \pi/4)(-1) + 0$

$= \left(\frac{\pi^2 - \pi}{4}\right) - 1 = \frac{-\pi^2 + \pi}{4} = \frac{\pi(-\pi + 1)}{4}$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right]$

$= \frac{\pi(-\pi + 1)}{4}.$

b) $\lim_{x \to \pi/2} \ln\left[\frac{\exp(3x^2 + 2x - 1)}{x + 1} \right]$

Solution

$\ln\left[\frac{\exp(3(\pi/2)^2 + 2(\pi/2) - 1)}{\pi/2 + 1} \right]$

$\ln\left[\frac{\exp(3\pi^2/4) + 2(\pi/2) - 1}{\pi/2 + 1} \right]$

$= \frac{\frac{3\pi^2}{4} + \pi - 1}{\frac{\pi + 2}{2}} = \frac{3\pi^2 + 4\pi - 4}{4_2} \times \frac{2}{\pi + 2}$

$= \frac{3\pi^2 + 4\pi - 4}{2(\pi + 2)} = \frac{(\pi/1 - 2/3)(\pi + 2)}{2(\cancel{\pi + 2})}$

$= \frac{3\pi - 2}{3} \times \frac{1}{2}$

$= \frac{3\pi - 2}{6}$

$\therefore \lim_{x \to \pi/2} \ln\left[\frac{\exp(3x^2 + 2x - 1)}{x + 1} \right] = \frac{3\pi - 2}{6}.$

c) $\lim_{x \to 2\sqrt{3}} \cos\left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})} \right]$

$= \cos\left[\sin^{-1} \frac{(2\sqrt{3} - 2)}{2\sqrt{3} - \sqrt{3}} \right]$

$= \cos\left[\sin^{-1} \sqrt{3}/2 \right]$

$= \cos 60$

$= 1/2.$

$\therefore \lim_{x \to \sqrt{3}} \cos\left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})} \right] = 1/2.$

d) $\lim_{x \to 4}\left(\frac{x^2-8x+16}{x^2-5x+4}\right)$

Solution

$\lim_{x \to 4}\left(\frac{2x-8}{2x-5}\right)$

$= \frac{2(4)-8}{2(4)-5} = 0/5 = 0$

$\therefore \lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = 0.$

No 2

a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

Solution

If we take our standard series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \cdots$

When $p = 2$ we get

$\frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \frac{2}{5^2} + \cdots$

Since

$\frac{2}{2\times3} < \frac{2}{2^2};\ \frac{2}{3\times4} < \frac{2}{3^2};\ \frac{2}{4\times5} < \frac{2}{4^2}$

Therefore, the given series is convergent.

b) $\frac{2r}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

Solution

Using standard series when $p = 2$

$\frac{2}{1^p} + \frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \cdots$

Since $p = 1$ the series converges.

Therefore, the given series is convergent.

c) $U_n = \frac{1+2n^2}{1+n}$

Solution

$\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{\left[1/n^2 + 2n^2/n^2\right]}{1/n^2 + n^2/n^2}$

$= \left[\frac{1/n^2 + 2}{1/n^2 + 1}\right]$

$= \frac{0+2}{0+1}$

$= 2$

$\lim_{n \to \infty} U_n = 2$

Therefore, the series is divergent.

No. 3

$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$

$|U_n| = \frac{x^n}{(2n+1)^3}$

$|U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$

$\frac{|U_{n+1}|}{|U_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x^n \cdot x}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$

$= \frac{x(2n+1)^3}{(2n+3)^3}$

$= \frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$

$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$

$= \frac{x\left[\frac{8n^3}{n^3} + \frac{12n^2}{n^3} + \frac{6n}{n^3} + \frac{1}{n^3}\right]}{\frac{8n^3}{n^3} + \frac{36n^2}{n^3} + \frac{54n}{n^3} + 27/n^3}$

$= \frac{x\left[8 + 12/n + 6/n^2 + 1/n^3\right]}{8 + 36/n + 54/n^2 + 27/n^3}$

$= \frac{x[8+0+0+0]}{8+0+0+0} = 8x/8 = x.$

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No. 4

Evaluate using L'Hopital's rule

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Solution

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$\lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$\lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$\lim_{x \to 0} \left[\frac{-\cos x + (-\sin x)}{6} \right]$$

$$\lim_{x \to 0} \left[-\frac{\cos x - \sin x}{6} \right]$$

$$= \frac{-1+0}{6} = -1/6$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^2} \right] = -1/6 .$$