ADIOTOMRE WILLIAM
PETROLEUM ENGINEERING
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The parametric equations of a curve are given in Equation (1) and (2).

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

In terms of t, determine

i an expression for the radius of curvature (R). and

ii expression for the co-ordinates (h, k) of the centre of curvature

Soln

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$

$\frac{dx}{dt} = t\cos t$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t$

$\frac{dy}{dt} = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx} = t\sin t \times \frac{1}{t\cos t} = \frac{\sin t}{\cos t}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \quad \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right)\frac{dt}{dx}$

$\Rightarrow \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2} = \frac{V\frac{du}{dt} - U\frac{dv}{dt}}{V^2}$

let $y = \sin t$

$x = \cos t$

$\frac{dy}{dt} = \cos t$

$dx/dt = -\sin t$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t - (-\sin^2 t)}{\cos^2 t} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \frac{1}{t\cos t}$$

Recall from trigonometric identities that: $\sin^2 t + \cos^2 \theta = 1$

$$= \frac{d^2y}{dx^2} = \frac{1}{t\cos^2 t}$$

Since $R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

$$R = \left[1 + \left(\frac{\sin t}{\cos t}\right)^2\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \times \frac{t\cos^3 t}{1}$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t\cos^3 t$$

$$R = \frac{t\cos^3 t}{\cos^3 t}$$

$R = t$

ii (h, k)

Recall ; $h = x_1 - R\sin\theta$ ...... (1)

$k = y_1 - R\cos\theta$ ..... (2)

$R = t$; $\theta = t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

Substituting for θ, x, y & R in equation (1) and (2)

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

The expression for the coordinate (h, k) at the centre of curvature is $(\cos t, \sin t)$