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Course ENG 281.

Assignment. 1

1 a) $$\lim_{x \to \pi/2} \left[\frac{\left(x^2 - \frac{\pi}{4}\right)\left(\sin(\cos x)\right)}{x - \pi/2} \right]$$

Solution.

For the numerator using product rule.

let $u = x^2 - \pi/4$ and $v = \sin(\cos x)$

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

For $u = x^2 - \pi/4$

$$\frac{du}{dx} = 2x$$

For $v = \sin(\cos x)$

let $w = \cos x$ and $v = \sin w$

$$\frac{dw}{dx} = -\sin x \qquad \frac{dv}{dw} = \cos w$$

$$\frac{dv}{dx} = \frac{dw}{dx} \times \frac{dv}{dw} = -\sin x \cos w$$

$$\frac{dv}{dx} = -\sin(\cos(\cos x))$$

$$\frac{dy}{dx} = \left[x^2 - \pi/4\right] \cdot -\cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$$

$$\frac{dy}{dx} = \left[x^2 - \pi/4\right] \cdot -\cos(\cos x)\sin x + \sin(\cos x) \cdot 2x$$

For the denominator, let $b = x - \pi/2$

$$\frac{db}{dx} = 1$$

$$\therefore \frac{\left(x^2-\frac{\pi}{4}\right)\left(-\cos(\cos x)\sin x\right)+\sin(\cos x)\,2x}{1}$$

$$\lim_{x\to \pi/2} \left(\frac{\pi^2}{2^2}-\frac{\pi}{4}\right)\left(-\cos\left(\cos\frac{\pi}{2}\right)\sin\frac{\pi}{2}\right)+\sin\left(\cos\pi/2\right)\,2\pi/2$$

$$=\left(\frac{\pi^2}{4}-\frac{\pi}{4}\right)\left(-1\right)+0$$

$$=\left(\frac{\pi^2-\pi}{4}\right)-1 = \frac{-\pi^2+\pi}{4}$$

$$=\frac{\pi(-\pi+1)}{4}$$

$$\therefore \lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)\sin(\cos x)}{x-\pi/2}\right]=\frac{\pi(-\pi+1)}{4}$$

c)

b) $$\lim_{x\to\frac{\pi}{2}} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$$

$$=\ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$$

d)

$$=\lim_{x\to\frac{\pi}{2}}\left[\frac{3\left(\frac{\pi}{2}\right)^2+\cancel{2}\left(\frac{\pi}{\cancel{2}}\right)-1}{\left(\frac{\pi}{2}\right)+1}\right]$$

$$=\frac{\frac{3\pi^2}{4}+\frac{\pi}{1}-\frac{1}{1}}{\frac{\pi+2}{2}} = \frac{3\pi^2+4\pi-4}{\cancel{4}\,2}\times\frac{\cancel{2}\,1}{\pi+2}$$

$$=\frac{3\pi^2+4\pi-4}{2(\pi+2)} = \frac{\left(\frac{\pi}{1}-\frac{2}{3}\right)\cancel{(\pi+2)}}{2\cancel{(\pi+2)}}$$

$$=\frac{3\pi-2}{3}\times\frac{1}{2}$$

$$= \frac{3\pi - 2}{6} \text{ or } \frac{\pi}{2} - \frac{1}{3}$$

$$\lim_{x \to \frac{\pi}{2}} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x + 1}\right)\right] = \frac{3\pi - 2}{6}$$

3) $$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\left(\frac{(2+\sqrt{3}) - 2}{2+\sqrt{3} - \sqrt{3}}\right)\right]$$

$$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \cos(60) = \frac{1}{2}$$

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$$

4) $$\lim_{x \to 4}\left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$$

$$= \lim_{x \to 4}\left[\frac{2x - 8}{2x - 5}\right]$$

$$= \lim_{x \to 4}\left[\frac{2(4) - 8}{2(4) - 5}\right] = \frac{8 - 8}{8 - 5}$$

$$= \frac{0}{3} = 0$$

$$\lim_{x \to 4}\left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right] = 0$$

2) Determine whether each of the following series is Convergent

a) $\frac{2}{2\times3}+\frac{2}{3\times4}+\frac{2}{4\times5}+\frac{2}{5\times6}+---$

Solution

$\boxed{Un}=$

To test the series $\frac{2}{2\times3}+\frac{2}{3\times4}+\frac{2}{4\times5}+\frac{2}{5\times6}+---$

If we take our standard series

$\frac{2}{2^p}+\frac{2}{3^p}+\frac{2}{4^p}+\frac{2}{5^p}+---$

when $P=2$, we get

$\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\frac{2}{5^2}+---$

Since

$\frac{2}{2\times3}<\frac{2}{2^2}$; $\frac{2}{3\times4}<\frac{2}{3^2}$; $\frac{2}{4\times5}<\frac{2}{4^2}$

Therefore, the given series is Convergent.

b) $\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+----$

Solution:

For Comparison test

Using Standard Series, when $P=2$

$\frac{2}{1^p}+\frac{2}{2^p}+\frac{2}{3^p}+\frac{2}{4^p}+---$

Since $P > 1$, the series Converges

Therefore, the given series is Convergent.

c) $U_n = \frac{1+2n^2}{1+n^2}$

Solution.

$$U_n = \frac{1+2n^2}{1+n^2}$$

$$\text{So} \lim_{n \to \alpha} U_n = \lim_{n \to \alpha} \frac{\left[\frac{1}{n^2} + \frac{2n^2}{n^2}\right]}{\frac{1}{n^2} + \frac{n^2}{n^2}}$$

$$= \left[\frac{\frac{1}{n^2} + \frac{2}{1}}{\frac{1}{n^2} + \frac{1}{1}}\right]$$

$$= \frac{0+2}{0+1}$$

$$= 2$$

$$\lim_{n \to \alpha} U_n = 2$$

Since $\lim_{n \to \alpha} U_n \neq 0$

Therefore, the series is divergent.

3) Find the range of values of x for which the series below is absolutely convergent

$$\frac{x}{27} + \frac{x^2}{125} + \cdots + \frac{x^n}{(2n+1)^3}$$

Soln

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{|U_{n+1}|}{|U_n|} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{\cancel{x^n} \cdot x}{(2n+3)^3} \times \frac{(2n+1)^3}{\cancel{x^n}}$$

$$= \frac{x(2n+1)^3}{(2n+3)^3}$$

$$= \frac{x(2n+1)(2n+1)(2n+1)}{(2n+3)(2n+3)(2n+3)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+36n^2+54n+27}$$

$$= \frac{x\left[\frac{8n^3}{n^3} + \frac{12n^2}{n^3} + \frac{6n}{n^3} + \frac{1}{n^3}\right]}{\frac{8n^3}{n^3} + \frac{36n^2}{n^3} + \frac{54n}{n^3} + \frac{27}{n^3}}$$

$$= \frac{x\left[8 + 12/n + 6/n^2 + 1/n^3\right]}{8 + 36/n + 54/n^2 + 27/n^3}$$

$$= \frac{x\,[\cancel{8} + 0 + 0 + 0]}{\cancel{8} + 0 + 0 + 0}$$

$$= \frac{\cancel{8}x}{\cancel{8}}$$

$$= x$$

For $\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right| = x$

For absolute convergence $\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right| < 1$

$\therefore$ Series convergent when $-1 \leq x \leq 1$

4) Evaluate using L'Hopital's Rule

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Soln

$$= \lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$= \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x + (-\sin x}{6} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6} \right]$$

$$= \frac{-1 + 0}{6}$$

$$= \frac{-1}{6}$$

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = \frac{-1}{6}$$