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15/ENG02/002
ENG 381 Assignment I

Solve the following

1) $\frac{d^2y}{dx^2} - dy/dx - 2y = 8$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

3) $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

4) $d^2y/dx^2 + 25y = 5x^2 + x$

5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

6) $d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$, given that at $x=0, y=1$ and $dy/dx = -2$

7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$

8) $d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

Solution

1) $d^2y/dx^2 - dy/dx - 2y = 8$

Auxillary equation: $m^2 - m - 2$

$GS = CF + PI$

CF: Solve LHS=0 $\therefore m^2 - m - 2 = 0$

$\therefore (m^2 - 2m + m - 2) = 0$

$m(m-2) + 1(m-2) \quad \therefore m = 2 \text{ or } -1$

$\therefore y = Ae^{-x} + Be^{2x}$

PI: $f(x) = 8$. Assume $y = C$

$\therefore dy/dx = 0$; $d^2y/dx^2 = 0$

Substituting dy/dx and d^2y/dx^2 in the given equation

$0 + -0 - 2C = 8$

$\frac{-2C}{} = 8$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$GS = CF + PI$

CF ÷ Solve LHS ∴ Auxillary equation $m^2 - 4 = 0$

$m^2 = 4 \quad \therefore m = \pm 2$

$\therefore y = A\cosh 2x + B\sinh 2x$

PI ÷ $f(x) = 10e^{3x}$ ∴ Assume $y = Ce^{3x}$

$dy/dx = 3Ce^{3x}$; $d^2y/dx^2 = 9Ce^{3x}$

Substitute d^2y/dx^2 and y in the given equation

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$C = 2 \quad \therefore y = 2e^{3x}$

$GS = CF + PI$

$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

3) $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

$GS = CF + PI$

CF ÷ Solve LHS=0 ∴ Auxillary equation $m^2 + 2m + 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - (4 \times 1 \times 1)}}{2 \times 1} = \frac{-2 \pm 0}{2}$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

PI ⇒ $f(x) = e^{-2x}$ ∴ Assume $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$; $d^2y/dx^2 = 4Ce^{-2x}$

Substitute the value of d^2y/dx^2 and dy/dx

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$GS = CF + PI$

$CF \Rightarrow$ solve LHS=0 $\therefore$ Auxillary equation $m^2 + 25 = 0$

$m^2 = -25 \quad \therefore m = \pm j5$

$\therefore y = A\cos 5x + B\sin 5x$

$PI: f(x) = 5x^2 + x \quad \therefore$ Assume $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

Substituted $\frac{d^2y}{dx^2}$ and y in the equation

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$x^2: \quad 25C = 5$

$C = \frac{1}{5}$

$x: \quad 25D = 1$

$D = \frac{1}{25}$

$x^0: \quad 2C + 25E = 0$

$2(\frac{1}{5}) + 25E = 0$

$25E = -\frac{2}{5}$

$E = -\frac{2}{125}$

$\therefore y = (\frac{1}{5} \times x^2) + (\frac{1}{25} \times x) + (-\frac{2}{125})$

$y = \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

$y = \frac{25x^2 + 5x - 2}{125}$

$GS = CF + PI$

$GS = A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$GS = CF + PI$

$CF \Rightarrow$ solve LHS=0 $\therefore$ Auxillary equation $m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0 \Rightarrow m(m-1) - 1(m-1) = 0$

$m = -1$ twice

$y = e^{-x}(A + Bx)$

$PI \Rightarrow f(x) = 4\sin x \quad \therefore$ Assume $y = C\cos x + D\sin x$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = -C\cos x - D\sin x$

Substitute into the general equation

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$(-D + 2C + D)\sin x + (-C - 2D + C)\cos x = 4\sin x$

$\sin x: \quad -D + 2C + D = 4$

$$\frac{2C}{2} = \frac{4}{2} \quad \therefore C = 2$$

$\cos x: \quad -C - 2D + C = 0$

$-2 - 2D + 2 = 0$

$-2D = 0$

$D = 0$

$y = 2\cos x + 0\sin x$

$y = 2\cos x$

$GS = CF + PI \quad \therefore GS = e^{-x}(A + Bx) + 2\cos x$

$d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$

$CF \Rightarrow$ Solve $LHS = 0 \quad m^2 + 4m + 5 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$$

$y = e^{-2x}(A\cos x + B\sin x)$

$PI \Rightarrow f(x) = 2e^{-2x} \quad \therefore y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}; \quad \frac{d^2y}{dx^2} = 4Ce^{-2x}$

Substitute in the equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$e^{-2x}: \quad 4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

$(GS),\ y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

Question 6 Continuation:

$x = 0 \; ; \; y = 1$

$1 = A + 2$

$A = -1$

$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$

$\frac{dy}{dx} = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$

If $x = 0$ and $dy/dx = -2$

$-2 = B + 2 - 4$

$B = 0$

Particular solution is $y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

CF $\Rightarrow$ LHS $= 0$ $\therefore$ Auxillary equation $= 3m^2 - 2m - 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{2 \pm \sqrt{(-2)^2 - (4 \times 3 \times -1)}}{2 \times 1}$

$= \frac{2 \pm \sqrt{4 + 12}}{2} = \frac{2 \pm 4}{2}$

$m = 3$ or -1

$\therefore y = Ae^{-x} + Be^{3x}$

PI $\div$ $f(x) = 2x - 3$ By $\cancel{t}$ $y = Cx + D$

$\frac{dy}{dx} = C$; $\frac{d^2y}{dx^2} = 0$

7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$ 8)

Note; It is a non-homogenous equation.

General Solution = Complementary Function + Particular integral

Complementary function.

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$$

the auxcillary equation becomes.

$$3m^2 - 2m - 1 = 0$$

$m = 1$ & $m = -1/3$ --- Real & different root.

$C.F = Ae^{x} + Be^{-1/3x}$

Particular integral.

Assumed P.I =

$$y = Cx + D \quad ---(i)$$

$$dy/dx = C \quad ---(ii)$$

$$d^2y/dx^2 = 0 \quad ---(iii)$$

∴ Sub equ(i), (ii) & (iii) into the general equation.

$$\therefore \; 3(0) - 2(C) - (Cx + D) = 2x - 3$$

$$0 - 2C - Cx - D = 2x - 3$$

Comparing co-efficient.

$$-Cx = 2x$$

$$-C = 2$$

$$C = -2 \quad ---(iv)$$

$$-2C - D = -3$$

Recall $C = -2$.

$$\therefore -2(-2) - D = 3$$

$$4 - D = 3$$

$$D = 4 - 3$$

$$D = 1 \quad ---(v)$$

$\therefore \; y = -2x +$

∴ General Solution =>

$$y = Ae^{x} + Be^{-1/3x} - 2x + 1$$

$d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$

It is a non-homogenous equation.

General Solution = Complementary Function + Particular integral.

Complementary function.

$$d^2y/dx^2 - 6\,dy/dx + 8y = 0$$

The auxillary equation becomes.

$$m^2 - 6m + 8 = 0$$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$m = 4$ & $m = 2$ (real & different root).

$\therefore C.F = Ae^{4x} + Be^{2x}$

Particular integral

Assumed P.I

$$y = Ce^{4x}$$

But since e^{4x} is both in the C.F & assumed P.I, we would multiply the P.I by x.

$$\therefore y = Cxe^{4x} \quad ---(i)$$

using Product rule.

$$dy/dx = C\{x4e^{4x} + e^{4x}\}. \quad ---(ii)$$

using Product rule.

$$d^2y/dx^2 = C\{x16e^{4x} + 4e^{4x} + 4e^{4x}\}$$

$$= C\{x16e^{4x} + 8e^{4x}\} \quad ---(iii)$$

Sub eqn(i), (ii) & (iii) into the general equation.

$$d^2y/dx^2 - 6\,dy/dx + 8y = 8e^{4x}$$

$$Cx16e^{4x} + C8e^{4x} - 6(Cx4e^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$$

$$Cx16e^{4x} + C8e^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$Cxe^{4x}(16 - 24 + 8) + Ce^{4x}(8 - 6) = 8e^{4x}$$

$$0 + Ce^{4x}(2) = 8e^{4x}$$

$$2C = 8$$

$$C = 8/2$$

$$C = 4$$

$\therefore y = Cxe^{4x}$

$y = 4xe^{4x}$

$\therefore$ General solution = Complementary function + Particular integral

$$y = Ae^{4x} + Be^{2x} + 4xe^{4x}$$