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MECHATRONICS
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1 Question

The parametric equation of a curve are given in Equation (1) and (2).

$$x = \cos t + t \sin t \quad \text{---(1)}$$
$$y = \sin t - t \cos t \quad \text{---(2)}$$

In term of t, determine

(i) an expression for the radius of curvature (R), and.

(ii) Expressions for the coordinates (h, k) of the centre of curvature.

Solution

(i)

$$y = \sin t - t\cos t.$$
$$x = \cos t + t \sin t.$$
$$\frac{dy}{dt} = \cos t - (\cos t - t \sin t)$$
$$\frac{dy}{dt} = \cancel{\cos t} - \cos t + t \sin t$$
$$\frac{dy}{dt} = t \sin t.$$

$$\frac{dx}{dt} = -\sin t + (t\cos t + \sin t)$$
$$\frac{dx}{dt} = -\sin t + t\cos t + \sin t$$
$$\frac{dx}{dt} = t\cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dt} = t\sin t \qquad \frac{dt}{dx} = \frac{1}{t\cos t}$$

$$\frac{dy}{dx} = \frac{t\sin t}{t\cos t} = \tan t \qquad \frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{t\cos t} = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t}$$

$$= \frac{1}{t\cos^3 t} = \frac{1}{t} \times \frac{1}{\cos^3 t} = t^{-1}\sec^3 t$$

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left(1 + (\tan t)^2\right)^{3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(1 + \tan^2 t)^{3/2}}{t^{-1}\sec^3 t}$$

$$(1 + \tan^2 t)^{3} = \sec^2 t$$

$$\therefore R = \frac{(\sec t)^{2 \times 3/2}}{t^{-1}\sec^3 t}$$

$$R = \frac{(\sec t)^3}{t^{-1}(\sec t)^3}$$

$$R = \frac{1}{t^{-1}} = t$$

$$R = t \text{ units.}$$

1) to find Centre of Curvature

$$x_1 = h + R\sin\theta$$

$$h = x_1 - R\sin\theta$$

$$k = Y_1 + R\cos\theta$$

$$\theta = \tan^{-1}\left\{\frac{dy}{dx}\right\}$$

$$\theta = \tan^{-1}(\tan t)$$

$$\theta = t$$

$$x_1 = \cos t + t\sin t$$

$$h = \cos t + t\sin t - (t)\sin t$$

$$h = \cos t + t\sin t - t\sin t$$

$$h = \underline{\cos t}.$$

$$k = Y_1 + R\cos\theta$$

$$Y_1 = \sin t - t\cos t$$

$$k = \sin t - t\cos t + (t)\cos t$$

$$k = \sin t - t\cos t + t\cos t.$$

$$k = \sin t.$$

Centre Of Curvature = $(\overset{h}{\cos t}, \overset{k}{\sin t})$