

Name: Jam Tertula Joekin  
Dept: Computer Engineering

Matric No: 15/ENG02/032

COURSE: ENG381 (Engineering Mathematics II) Assignments -

1)  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Note: It is a non-homogeneous equation.

∴ General Solution = Complementary Function + Particular Integral

∴ Complementary function (C.F.)

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

The auxiliary equation becomes

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + 1m - 2 = 0$$

$$m(m-2) + 1(m-2) = 0$$

$$m+1 = 0 \quad \& \quad m-2 = 0$$

$$m = -1 \quad \& \quad m = 2 \quad \dots \text{Real \& different roots}$$

$$\therefore \text{C.F.} = Ae^{-x} + Be^{2x}$$

Particular integral (P.I.)

$$y = C \dots (i)$$

$$\frac{dy}{dx} = 0 \dots (ii)$$

$$\frac{d^2y}{dx^2} = 0 \dots (iii)$$

Substituting eqn(i), (ii) \& (iii) into the general equation.

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$$

$$0 - 0 - 2(C) = 8$$

$$-2C = 8$$

$$C = 8/-2$$

$$C = -4$$

$$\therefore y = C$$

$$\therefore y = -4$$

General Solution = Complementary Function + Particular Integral.

$$y = Ae^{-x} + Be^{2x} + (-4)$$

$$y = Ae^{-x} + Be^{2x} - 4$$

$$d^2y/dx^2 - 4y = 10e^{3x}$$

Note: It is a non-homogenous equation

∴ General Solution = Complementary Solution + Particular Integral.

∴ Complementary function (C.F.)

$$d^2y/dx^2 - 4y = 0$$

The auxiliary equation becomes

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \pm \sqrt{4}$$

$$m = \pm 2$$

$$m_1 = 2 \quad m_2 = -2$$

$$\therefore \text{C.F.} = Ae^{2x} + Be^{-2x}$$

Particular integral

Assume P.I

$$y = ce^{3x} \dots (i)$$

$$dy/dx = 3ce^{3x} \dots (ii)$$

$$d^2y/dx^2 = 9ce^{3x} \dots (iii)$$

Sub eqs(i), (ii) & (iii) into the general equation.

$$\therefore d^2y/dx^2 - 4y = 10e^{3x}$$

$$\therefore 9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$$

$$9ce^{3x} - 4ce^{3x} = 10e^{3x}$$

$$e^{3x}(9c - 4c) = 10e^{3x}$$

$$9c - 4c = 10$$

$$5c = 10$$

$$c = 10/5$$

$$c = 2$$

$$\therefore y = ce^{3x}$$

$$y = 2e^{3x}$$

∴ General Solution = Complementary function + Particular Integral.

$$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$$



$$d^2y/dx^2 + 2 dy/dx + y = e^{-2x}$$

Note: It is a non-homogeneous equation.

∴ General Solution = Complementary Function + Particular Integral.

Complementary function (C.F.):

$$d^2y/dx^2 + 2 dy/dx + y = 0$$

The auxiliary equation becomes:

$$m^2 + 2m + 1 = 0$$

$$∴ m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$m = -1 \quad (\text{twice})$$

$$∴ \text{C.F.} = e^{-x} (A + Bx)$$

Particular integral

$$\text{Assumed P.I.} = y = Ce^{-2x} \quad \dots \text{(i)}$$

$$dy/dx = -2Ce^{-2x} \quad \dots \text{(ii)}$$

$$d^2y/dx^2 = 4Ce^{-2x} \quad \dots \text{(iii)}$$

Sub eqns (i), (ii) & (iii) into the general equation

$$d^2y/dx^2 + 2 dy/dx + y = e^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$∴ y = Ce^{-2x}$$

$$∴ y = e^{-2x}$$

∴ General Solution = Complementary Function + Particular Integral

$$y = e^{-x} (A + Bx) + e^{-2x}$$

$$4) \frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

Note: It is a non-homogenous equation

$\therefore$  General Solution = Complementary Function + Particular integral

$\therefore$  Complementary Function:

$$\frac{d^2y}{dx^2} + 25y = 0$$

The auxiliary equation becomes:

$$m^2 + 25 = 0$$

$$m^2 = -25$$

$$m = \pm \sqrt{-25}$$

$$m = \pm \sqrt{25} \cdot \sqrt{-1}$$

$$m = \pm 5j \quad \text{--- complex root}$$

Comparing with  $m = \alpha \pm \beta j$

$$\alpha = 0 \quad \beta = 5$$

$$\therefore \text{C.F.} = e^{0x} (A \cos 5x + B \sin 5x)$$

$$\text{Since } e^0 = 1$$

$$\therefore \text{C.F.} = A \cos 5x + B \sin 5x$$

Particular integral:

Assumed P.I

$$y = Cx^2 + Dx + E \quad \text{--- (i)}$$

$$\frac{dy}{dx} = 2Cx + D \quad \text{--- (ii)}$$

$$\frac{d^2y}{dx^2} = 2C \quad \text{--- (iii)}$$

Sub eqn(i), (ii) & (iii) into the general equation:

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$$

$$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$$

$$25Cx^2 = 5x^2$$

$$25C = 5$$

$$C = \frac{5}{25}$$

$$C = \frac{1}{5} \quad \text{--- (iv)}$$

$$25Dx = x$$

$$25D = 1$$

$$D = \frac{1}{25} \quad \text{--- (v)}$$



Cont'd

$$4) \quad 2C + 25E = 0$$

$$\text{Recall } C = \frac{1}{5}$$

$$2\left(\frac{1}{5}\right) + 25E = 0$$

$$\frac{2}{5} + 25E = 0$$

$$25E = -\frac{2}{5}$$

$$E = -\frac{2}{5} \times \frac{1}{25}$$

$$E = -\frac{2}{125} \quad \text{--- (vii)}$$

∴ Sub eqn (iv), (v) & (vii) into eqn (i):

$$y = Cx^2 + Dx + E$$

$$y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

∴ General solution = Complementary Function + Particular Integral

$$y = A \cos 5x + B \sin 5x + \left(\frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}\right)$$

$$\therefore y = A \cos 5x + B \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$$

$$5) \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

6)

Note: It is a non-homogeneous equation

$$\text{General Solution} = \text{Complementary Function} + \text{Particular Integral}$$

∴ Complementary function:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

The auxiliary equation becomes

$$m^2 - 2m + 1 = 0$$

$$m^2 - m - m + 1 = 0$$

$$m(m-1) - 1(m-1) = 0$$

$$m = 1 \text{ (twice)}$$

$$\therefore \text{C.F.} = e^x (A + Bx)$$

Particular integral:

Assumed P.I =

$$y = C\cos x + D\sin x \text{ --- (i)}$$

$$\frac{dy}{dx} = -C\sin x + D\cos x \text{ --- (ii)}$$

$$\frac{d^2y}{dx^2} = -C\cos x - D\sin x \text{ --- (iii)}$$

Sub eqns, (i), (ii) & (iii) into the general equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$$

$$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$$

$$\therefore -C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$$

$$\Rightarrow 2C\sin x - 2D\cos x = 4\sin x$$

Comparing co-efficient:

$$2C = 4$$

$$-2D = 0$$

$$C = 4/2 \text{ --- (iv)}$$

$$D = 0 \text{ --- (v)}$$

$$\therefore y = C\cos x + D\sin x$$

$$y = 2\cos x + 0\sin x$$

$$y = 2\cos x \text{ --- (vi)}$$

∴ General Solution = Complementary Function + Particular integral

$$y = e^x (A + Bx) + 2\cos x$$



$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$$

Given that at  $x=0$ ,  $y=1$  &  $\frac{dy}{dx} = -2$

Note: It is a non-homogeneous equation

General Solution = Complementary Function + Particular Integral

∴ Complementary Function:-

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

The auxiliary equation becomes:

$$m^2 + 4m + 5 = 0$$

Using quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=1, b=4, c=5$$

$$\frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times 5)}}{2 \times 1}$$

$$\Rightarrow \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$$\sqrt{-4} = \sqrt{4} \times \sqrt{-1}$$

$$= 2j$$

$$\therefore \frac{-4 \pm 2j}{2}$$

$$= -2 \pm j$$

Comparing with  $m = \alpha + \beta j$   
 $\alpha = -2$   $\beta = 1$

$$y = e^{\alpha x} (A \cos \beta x + B \sin \beta x)$$

$$y = e^{-2x} (A \cos x + B \sin x)$$

∴ Particular integral

Assumed P.I

$$y = Ce^{-2x}$$

But since  $Ce^{-2x}$  is both in the C.F. & assumed P.I, we would multiply the P.I by  $x$ .



$$\therefore y = x C e^{-2x}$$

$$y = C x e^{-2x} \quad \text{--- (i)}$$

$$dy/dx = -2 C x e^{-2x} \quad \text{--- (ii)}$$

$$d^2y/dx^2 = 4 C x e^{-2x} \quad \text{--- (iii)}$$

Sub eqs (i), (ii) & (iii) into the general equation

$$\therefore d^2y/dx^2 + 4 dy/dx + 5y = 2e^{-2x}$$

$$4 C x e^{-2x} + 4(-2 C x e^{-2x}) + 5(C x e^{-2x}) = 2e^{-2x}$$

$$4 C x e^{-2x} - 8 C x e^{-2x} + 5 C x e^{-2x} = 2e^{-2x}$$

$$C x e^{-2x} (4 - 8 + 5) = 2e^{-2x}$$

$$C x e^{-2x} (1) = 2e^{-2x}$$

$$C x = 2$$

$$C = 2/x \quad \text{--- (iv)}$$

Sub eq (iv) into (i)

$$y = C x e^{-2x}$$

$$y = \frac{2}{x} x e^{-2x}$$

$$\therefore y = 2e^{-2x}$$

The general solution becomes:

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

To find the values of A & B

$$\text{when } x=0, y=0$$

$$1 = e^{-2(0)} (A \cos 0 + B \sin 0) + 2e^{-2(0)}$$

$$1 = 1 (A \cdot 1 + B \cdot 0) + 2$$

$$1 = A + 2$$

$$A = 1 - 2$$

$$A = -1$$

$$\text{when } x=0 \quad dy/dx = -2$$

$$y = e^{-2x} (A \cos x + B \sin x) + 2e^{-2x}$$

$$y = e^{-2x} A \cos x + e^{-2x} B \sin x + 2e^{-2x}$$

using product rule

$$dy/dx = -e^{-2x} [A \sin x] - 2e^{-2x} [A \cos x] + e^{-2x} [B \cos x]$$

$$- 2e^{-2x} [B \sin x] - 4e^{-2x}$$

$$-2 = -e^{-2(0)} [A \sin 0] - 2e^{-2(0)} [A \cos 0] + e^{-2(0)} [B \cos 0]$$

$$-2e^{-2(0)} [B \sin 0] - 4e^{-2(0)}$$



Cont'd.

$$G) -2 = 0 - 2[A] + B - 0 - 4$$

$$-2 = -2A + B - 4$$

Re-arranging.

$$-2A + B = -2 + 4$$

But recall

$$A = -1$$

$$\therefore -2(-1) + B = 2$$

$$2 + B = 2$$

$$B = 2 - 2$$

$$B = 0$$

Sub  $A = -1$  &  $B = 0$  into the equation.

$$y = e^{-2x} (-1 \cos x + 0 \sin x) + 2e^{-2x}$$

$$y = e^{-2x} (-\cos x) + 2e^{-2x}$$

$$y = e^{-2x} (2 - \cos x)$$

$$7) 3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 2x - 3$$

8

Note: It is a non-homogeneous equation.

General Solution = Complementary Function + Particular Integral

Complementary function:

$$3 \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 0$$

The auxiliary equation becomes:

$$3m^2 - 2m - 1 = 0$$

$m = 1$  &  $m = -1/3$  --- Real & different roots.

$$C.F. = Ae^x + Be^{-1/3x}$$

Particular integral:

Assumed P.I. =

$$y = Cx + D \quad \dots (i)$$

$$\frac{dy}{dx} = C \quad \dots (ii)$$

$$\frac{d^2y}{dx^2} = 0 \quad \dots (iii)$$

$\therefore$  Sub eqns (i), (ii) & (iii) into the general equation:

$$\therefore 3(0) - 2(C) - (Cx + D) = 2x - 3$$

$$0 - 2C - Cx - D = 2x - 3$$

Comparing Co-efficient:

$$-Cx = 2x$$

$$-C = 2$$

$$C = -2 \quad \dots (iv)$$

$$-2C - D = -3$$

Recall  $C = -2$ :

$$\therefore -2(-2) - D = -3$$

$$4 - D = -3$$

$$D = 4 - 3$$

$$D = 1 \quad \dots (v)$$

$$\therefore y = -2x + 1$$

$$\therefore \text{General Solution} \Rightarrow y = Ae^x + Be^{-1/3x} - 2x + 1$$

2



$$8) \frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 8y = 8e^{4x}$$

It is a non-homogeneous equation.

General Solution = Complementary Function + Particular Integral.

Complementary Function:

$$\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 8y = 0$$

The auxiliary equation becomes:

$$m^2 - 6m + 8 = 0$$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$$m=4 \quad \& \quad m=2 \quad (\text{real \& different roots})$$

$$\therefore \text{C.F.} = Ae^{4x} + Be^{2x}$$

Particular Integral

Assumed P.I.

$$y = Ce^{4x}$$

But since  $e^{4x}$  is both in the C.F. & assumed P.I., we would multiply the P.I. by  $x$ .

$$\therefore y = Cx e^{4x} \quad \text{--- (i)}$$

using Product rule.

$$\frac{dy}{dx} = C \{ x \cdot 4e^{4x} + e^{4x} \} \quad \text{--- (ii)}$$

$$\therefore y_2 = C_2 x e^{4x}$$

$$y = 4x e^{4x}$$

$\therefore$  General Solution = Complementary Function + Particular integral

$$y = A e^{4x} + B e^{2x} + 4x e^{4x}$$