2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$Gs = Cf + PI$

Cf = Solve Lts $\therefore$ Auxiliary Equation $= m^2 - 4 = 0$

$m^2 = 4$ $\therefore m = \pm 2$

$\therefore y = A\cosh 2x + B\sinh 2x$

$PI \div f(x) = 10e^{3x}$ $\therefore$ Assume $y = Ce^{3x}$

$dy/dx = 3Ce^{3x}$; $d^2y/dx^2 = 9Ce^{3x}$

Substitute d^2y/dx^2 and y in the given equation.

$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$

$5C\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$5C = 10$

$C = 2$ $\therefore y = 2e^{3x}$

$Gs = Cf + PI$

$y = A\cosh 2x + B\sinh 2x + 2e^{3x}$

3 $d^2y/dx^2 + 2\,dy/dx + y = e^{-2x}$

$Gs = Cf + PI$

Cf = Solve $Lts = 0$ Auxiliary equation $= m^2 + 2m + 1 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)}}{2 \times 1} = \frac{-2 \pm 0}{2}$

$m = -1$ twice

$y = e^{-x}(A + Bx$

$PI \Rightarrow f(x) = e^{-2x}$ $\therefore$ Assume $y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}$; $d^2y/dx^2 = 4Ce^{-2x}$

Substitute the value of d^2y/dx^2 and dy/dx

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x})$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$ $\therefore y = e^{-2x}$

$Gs = Cf + PI$

$y = e^{-x}(A + Bx) + e^{-2x}$

4 $d^2y/dx^2 + 25y = 5x^2 + x$

$Gs = Cf + PI$

Cf = Solve $Lts = 0$ Aux equation $m^2 + 25 = 0$

$m^2 = -25$ $\therefore m = \pm i5$

$\therefore y = A\cos 5x + B\sin 5\theta$

$PI \div f(x) = 5x^2 + x$ $\therefore$ Assume $y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2(x + D)$

$\frac{d^2y}{dx^2} = 2C$

Substitute d^2y/dx^2 and y in the equation.

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$x^2 : 25C = 5$

$C = 1/5$

$x : 25D = 1$

$D = 1/25$

$x^0 : 2C + 25E = 0$

$2(1/5) + 25E = 0$

$25E = -2(1/5)$

$E = -2/125$

$\therefore y = (1/5 \times x^2) + (1/25 \times x) + (-2/125)$

$y = x^2/5 + x/25 - 2/125$

$y = \frac{25x^2 + 5x - 2}{125}$

$GS = CF + PI$

$GS = A\cos 5x + B\sin 5x + \frac{25x^2 + 5x - 2}{125}$

5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

$GS = CF + PI$

$CF \Rightarrow$ solve $RHS = 0$ $\therefore$ Auxiliary equation $m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0 \Rightarrow m(m-1) - 1(m-1) = 0$

$m = 1$ twice

$y = e^{-x}(A + Bx)$

$PI \Rightarrow f(x) = 4\sin x$ $\therefore$ Assume $y = (C\cos x + D)\sin x$

$dy/dx = -C\sin x + D\cos x$

$d^2y/dx^2 = -(C\cos x - D)\sin x$

Substitute into general equation.

$-(C\cos x) - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = 4\sin x$

$-C\cos x - D\sin x + 2C\sin x - 2D\cos x + (C\cos x + D)\sin x = 4\sin x$

$(-D + 2C + D)\sin x + (-C - 2D + C)\cos x = 4\sin x$

$\sin x: \quad -D + 2(-1D) = 4$

$\frac{2C}{2} = \frac{4}{2} \quad \therefore C = 2$

$\cos x: \quad -C - 2D + C = 0$

$-2 - 2D + 2 = 0$

$-2D = 0$

$D = 0$

$y = 2\cos x + 0 \sin x$

$y = 2\cos x$

$GS = CF + PI \quad \therefore GS = e^{-x}(A + Bx) + 2\cos x$

b) $d^2y/dx^2 + 4\, dy/dx + 5y = 2e^{-2x}$

$CF \Rightarrow$ Solve lets $= 0$ $\qquad m^2 + 4m + 5 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad \frac{-4 \pm \sqrt{4^2 - 4(5)}}{2} = \frac{-4 \pm \sqrt{-4}}{2}$

$\frac{-4 \pm 2i}{2} = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

$PI \Rightarrow f(x) = 2e^{-2x} \quad \therefore y = Ce^{-2x}$

$dy/dx = -2Ce^{-2x}; \quad \frac{d^2y}{dx^2} = 4Ce^{-2x}$

Substitute in the equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$e^{-2x}: \quad 4C - 8C + 5C = 2$

$C = 2$

$y = 2e^{-2x}$

$(GS)\ y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$x = 0 \ ; \ y = 1$

$1 = A + 2$

$A = -1$

$y = e^{-2x}(-\cos x + B\sin x) + 2e^{-2x}$

$dy/dx = e^{-2x}(\sin x + B\cos x) - 2e^{-2x}(-\cos x + B\sin x) - 4e^{-2x}$

If $x = 0$ and $dy/dx = -2$

$-2 = B + 2 - 4$

$B = 0$

Solution: $y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

Scanned by CamScanner

7. $3d^2y/dx^2 - 2\, dy/dx - y = 2x-5$

$CF = LHS = 0$ $\therefore$ Auxiliary equation $= 3m^2 - 2m + 1 = 0$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\quad \therefore \dfrac{-2 \pm \sqrt{(-2)^2 - 4(3)}}{\cancel{2} \times 1}$

$= \dfrac{2 \pm \sqrt{4+12}}{2} \quad = \dfrac{2 \pm 4}{2}$

$m = 3$ or -1

$\therefore y = Ae^{-x} + Be^{3x}$

$PI: f(x) = 2x - 5$ $\quad$ But $y = (x + D)$

$\dfrac{dy}{dx} = C \; ; \; \dfrac{d^2y}{dx^2} = 0$

Substitute into the given equation.

$3(0) - 2(C) - (C(x+D)) = 2x - 3$

$-2C - (x - D) = 2x - 3$

$x: \quad -C = 2$

$C = 2$

$x^0 = 2C - D = -3$

$-2(-2) - D = -3$

$4 - D = -3$

$7 = D$

$\therefore y = -2x + 7$

$G_s = CF + \underline{PI}$

$G_s = Ae^{-x} + Be^{3x} - 2x + \underline{7}$

5. $d^2y/dx^2 - 6\, dy/dx + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$ — auxiliary equation

$CF \Rightarrow LHS = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m - 4) = 0$

$m = 4$ or 2

$y = Ae^{2x} + Be^{4x}$

$PI \Rightarrow f(x) = 8e^{4x}$ $\quad$ But $y = Ce^{4x}$

$\dfrac{dy}{dx} = 4Ce^{4x} \; ; \; \dfrac{d^2y}{dx^2} = 16Ce^{4x}$ (sub. $\frac{d^2y}{dx}$ into general

$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$

$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$e^{4x} = 16C - 24C + 8C = 8$