INAIBO E ISAIAH/16/ENG04/026
Elect/Elect Engineering
ENG 281 Assignment

Question

The parametric equations of a curve are given in equations (1) and (2).

$$x = \cos t + t\sin t \quad \text{---} (1)$$

$$y = \sin t - t\cos t \quad \text{---} (2)$$

In terms of t, determine:

(i) an expression for the radius of curvature (R)

(ii) expressions for the coordinates (h, k) of the centre of curvature

Soln

(i) Recall that:

$$\text{Radius of Curvature } (R) = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$x = \cos t + t\sin t$$

$$\frac{dx}{dt} = -\sin t + (t\cos t + (1)\sin t) = t\cos t$$

$$y = \sin t - t\cos t$$

$$\frac{dy}{dt} = \cos t - (-t\sin t + (1)\cos t) = \cos t + t\sin t - \cos t$$

$$\frac{dy}{dt} = t\sin t$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt} \times \frac{1}{\frac{dx}{dt}}$$

Recall $\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

$$\therefore \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) = \frac{\cos t \cdot \cos t - \sin t(-\sin t)}{(\cos t)^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t}$$