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Assignment

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Sol^n

$CF = m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2$ or -1

$y = Ae^{-x} + Be^{2x}$

Assumed PI; $y = C$ ... (1)

$\frac{dy}{dx} = 0$ ... (2)

$\frac{d^2y}{dx^2} = 0$ ... (3)

Substituting eqn (1), (2) and (3) into the original equation.

$0 - 0 - 2(C) = 8$

$-2C = 8$

$C = -4$

General solution

CF + PI

$y = Ae^{-x} + Be^{2x} - A$

2) $\frac{d^2y}{dx^2} - Ay = 10^{3x}$

$CF = m^2 - A = 0$

$(m-2)(m+2) = 0$

$m = 2$ or -2

$y = Ae^{2x} + Be^{-2x}$

Assumed PI: $y = Ce^{kx}$

$y = Ce^{3x}$ ... (1)

$$\frac{dy}{dx} = 3ce^{3x} \quad \cdots (2)$$

$$\frac{d^2y}{dx^2} = 9ce^{3x} \quad \cdots (3)$$

Sub eqn 1, 2 & 3 into the original equation

$$9ce^{3x} - 4(ce^{3x}) = 10e^{3x}$$

$$ce^{3x}(9-4) = 10e^{3x}$$

$$5ce^{3x} = 10e^{3x}$$

$$C = 2$$

General solution

$$y = CF + PI$$

$$y = Ae^{2x} + Be^{-2x} + 2e^{3x}$$

3) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

Soln

CF: $m^2 + 2m + 1 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{0}}{2} = \frac{-2 \pm 0}{2}$$

~~m = -1 twice~~ $m = -1$ twice

$$y = e^{-x}(A + Bx)$$

assumed PI: $y = Ce^{-2x} \quad \cdots (1)$

$$\frac{dy}{dx} = -2Ce^{-2x} \quad \cdots (2)$$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x} \quad \cdots (3)$$

sub eqn (1), (2) and (3) into the original eqn

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 2Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$3Ce^{-2x} = e^{-2x}$$

$$C = 1/3$$

assumed PI ; $y = \frac{1}{3}e^{-2x}$

General solution: CF + PI

$$y = e^{-x}(A+Bx) + \frac{1}{3}e^{-2x}$$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Sol^n

CF : $m^2 + 25 = 0$

$(m-5)(m+5)$

$m = -5$ or 5

$y = Ae^{-5x} + Be^{5x}$

Assumed PI: $y = Cx^2 + Dx + E$ .. (1)

$\frac{dy}{dx} = 2Cx + D$ .. (2)

$\frac{d^2y}{dx^2} = 2C$ ... (3)

Substituting eq^n (1), (2) and (3) in the original equation

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

Equating coefficient;

$25Cx^2 = 5x^2$

$25C = 5$

$C = \frac{1}{5}$

$25D = 1$

$D = \frac{1}{25}$

$2C + 25E = 0$

$2(\frac{1}{5}) + 25E = 0$

$\frac{2}{5} + 25E = 0$

$25E = -\frac{2}{5}$

$E = -\frac{2}{125}$

$y = \frac{1}{5}Cx^2 + \frac{1}{25}x - \frac{2}{125}$

General solution; CF + PI

$y = Ae^{-5x} + Be^{5x} + \frac{1}{5}Cx^2 + \frac{1}{25}x - \frac{2}{125}$

5) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

Sol^n

CF: $m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1)$

$m = 1$ twice

$y = e^x(A + Bx)$

assumed PI: $C\cos x + D\sin x$

$\therefore y = C\cos x + D\sin x \quad \cdots (1)$

$dy/dx = -C\sin x + D\cos x \quad \cdots (2)$

$d^2y/dx^2 = -C\cos x - D\sin x \quad \cdots (3)$

substituting eqn (1), (2) and (3) into the original equation

$-C\cos x - D\sin x - 2(-C\sin x + D\cos x) + C\cos x + D\sin x = A\sin x$

$-\cancel{C\cos x} - \cancel{D\sin x} + 2C\sin x - 2D\cos x + \cancel{C\cos x} + \cancel{D\sin x} = A\sin x$

$2C\sin x - 2D\cos x = A\sin x$

Equating coefficient: $2C = A \qquad \therefore C = 2$

$-2D = 0 \qquad , D = 0$

$y = 2\cos x + 0\sin x \qquad = 2\cos x$

General Solution: $y = CF + PI$

$y = e^x(A + Bx) + 2\cos x$

6) $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 5y = 2e^{-2x}$, given that $x = 0$, $y = 1$ and $dy/dx = -2$

Sol^n

$CF = m^2 + 4m + 5 = 0$

$m + 5m - m + 5 = 0$

$(m+5)(m-1) = 0$

$m = -5$ or 1

$y = Ae^x + Be^{-5x}$

Assumed PI: $y = Ce^{-2x} \quad \cdots (1)$

$dy/dx = -2Ce^{-2x} \quad \cdots (2)$

$d^2y/dx^2 = 4Ce^{-2x} \quad \cdots (3)$

Substituting eqn (1), (2) & (3) into the original equation

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$\therefore y = 2e^{-2x}$

General solution: PI + CF

$= Ae^{x} + Be^{-5x} + 2e^{-2x}$

when $y = 1$, $x = 0$, $dy/dx = -2$

when $y = 1$, $x = 0$ into $y = Ae^{x} + Be^{-5x} + 2e^{-2x}$

$1 = Ae^{0} + Be^{5(0)} + 2e^{-2(0)}$

$1 = A + B + 2$

$A + B = -1 \quad \ldots (1)$

$\frac{dy}{dx} = Ae^{x} - 5Be^{-5x} - 4e^{-2x}$

when $dy/dx = -2$, $x = 0$

$-2 = Ae^{(0)} - 5Be^{-5(0)} - 4e^{-2(0)}$

$-2 = A - 5B - 4$

$2 = A - 5B \quad (2)$

solving eqn (1) and (2) simultaneously

$-6B = 3$

$B = -1/2$

$\therefore A - 5(-1/2) = 2$

$A + 5/2 = 2$

$A = -1/2$

$\therefore y = -\frac{1}{2}e^{x} - \frac{1}{2}e^{-5x} + 2e^{-2x}$

7) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

soln

CF: $3m^2 - 2m - 1 = 0$

$3m^2 - 3m + m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$M = 1$ or $-1/3$

$Ae^{x} + Be^{-1/3x}$

Assumed PI; $y = Cx + D$ ... (1)

$$\frac{dy}{dx} = C \quad \text{... (2)}$$

$$\frac{d^2y}{dx^2} = 0 \quad \text{... (3)}$$

Substituting eqn (1) (2) and (3) in the original equation

$$3(0) - 2(C) - (Cx + D) = 2x - 3$$

$$-2C - Cx - D = 2x - 3$$

Equating coefficient; $-C = 2$

$$C = -2$$

$$-2C - D = -3$$

$$-2(-2) - D = -3$$

$$4 - D = -3$$

$$-D = -7$$

$$D = 7$$

$$y = -2x + 7$$

General solution; $y = CF + PI$

$$y = Ae^{x} + Be^{-1/3x} - 2x + 7$$

8) $$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

Soln

CF: $m^2 - 6m + 8 = 0$

$$m^2 - 4m - 2m + 8 = 0$$

$$m(m-4) - 2(m-4) = 0$$

$$(m-2)(m-4) = 0$$

$$m = 2 \text{ or } 4$$

$$y = Ae^{2x} + Be^{4x}$$

Assumed PI; $y = Ce^{4x}$ ... (1)

$$\frac{dy}{dx} = 4Ce^{4x} \quad \text{... (2)}$$

$$\frac{d^2y}{dx^2} = 16Ce^{4x} \quad \text{... (3)}$$

substituting eqn (1), (2) and (3) into the original equation

$$16Ce^{4x} - 6(4Ce^{4x}) + 8(Ce^{4x}) = 8e^{4x}$$

$$16Ce^{4x} - 24Ce^{4x} + 8Ce^{4x} = 8e^{4x}$$

~~16e~~ divide through by e^{4x}

$$16C - 24C + 8C = 8$$

$$0 = 8$$

$$\therefore c = 0$$

$$y = 0e^{4x} = 0$$

General solution; $y = CF + PI$

$$y = Ae^{2x} + Be^{4x} + 0$$

$$y = Ae^{2x} + Be^{4x}.$$