Name: Victor Ocheni Ugbedeojo
Reg. No: 16/ENG04/087
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Dept: Electrical/Electronics

1 The parametric equation of a curve are as given in equation ① and ②

$x = \cos t + \sin t$ . . . ①

$Y = \sin t - t\cos$ . . . ②

in terms of t, determine

① an expression for the radius of curvature (R) and

ii. expression for the coordinates (h, j, k) of the center of curvature.

i Taking R as radius of curvature.

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

we take $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$; $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$

$\frac{dy}{dx} = \cos t - (-\sin t + \cos t)$ (from equ ② $y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t = t\sin t$

from equation ① $x = \cos t + t\sin t$

$\frac{dx}{dt} = -\sin t + (t \times \cos t = 1 \times \sin t)$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$

$\frac{dt}{dx} = \frac{1}{t\cos t}$ . . . . . ④

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = t\sin t \times \frac{1}{t\cos t} = \frac{\sin t}{\cos t}$ [recall $\frac{\sin\theta}{\cos t} = t$

$\frac{dy}{dt} = \tan t$

for $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$ but no derivative for x

$\therefore \frac{d}{dt}\left[\frac{dy}{dx}\right]\frac{dt}{dx}$ [from equ ④ $\frac{dt}{dx} = \frac{1}{t\cos t}$

$\therefore \frac{d}{dt}\left[\tan t\right] \times \frac{1}{t\cos t} = \sec^2 t \times \frac{1}{t\cos t}$

(because $\frac{dy}{dx} \tan t = \sec^2 \theta$)

$\frac{d^2y}{dx^2} = \frac{\sec^2 x}{t\cos t}$

$\therefore \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \div t\cos t = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$

then $R = \left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2} = \frac{(1+\tan^2 t)^{3/2}}{\frac{1}{t\cos^3 t}}$

recall $1+\tan^2\theta = \sec^2\theta$ $\quad \frac{(\sec^2 t)^{3/2}}{\frac{1}{t\cos^3 t}} = \frac{\left(\sqrt{\sec^2 t}\right)^3}{\frac{1}{t\cos^3 t}}$

$R = \frac{\sec^3 t}{\frac{1}{t\cos^3 t}}$ $\quad \therefore \sec^3 t \times t\cos^3\theta = \frac{1}{\cos^3 t} \times \frac{t\cos^3 t}{1}$

$\therefore R = t$

but $\tan\theta = \frac{dy}{dx}$ $\quad \therefore \tan\theta = \tan t \quad \therefore \theta = t$
(angle of slope)

5) $h = x_1 - R\sin t$
from equ I $\quad x = \cos t + t\sin t \qquad R_1 = t$
$h = \cos t + t\sin t - t\sin t$
$h = \cos t$

) $K = y_1 + R\cos t$ (from eqn(2) $y_1 = \sin t - t\cos t$)
$K = \sin t - t\cos t + t\cos t$
$K = \sin t$