3

$U_n = \dfrac{x^n}{(2n+1)^3}$, $U_{n+1} = \dfrac{x^{n+1}}{(2n+2)^3}$

$\lim \dfrac{U_{n+1}}{U_n} = \dfrac{x^{n+1}}{(2n+2)^3} \times \dfrac{(2n+1)^3}{x^n}$

$\dfrac{x(2n+1)^3}{(2n+2)^3} = \dfrac{8n^3 + 12n^2 + n + 1}{8n^3 + 24n^2 + 24n + 8}$

divide by n^3

$= \dfrac{(8 + 2/n + 1/n^2 + 1/n^3)}{(8 + 12/n + 24/n^2 + 8/n^3)}$

as $n \to \infty$

$1/n \to 0$

$\dfrac{8x}{8} \geq x - 1$

$x < 1$

4

$\lim_{x \to 0} \left[\dfrac{\sin x - \cos x}{x^3} \right]$

by using l'hopital's rule

$y = \left[\dfrac{\sin x - \cos x}{x^3} \right]$

$\dfrac{dy}{dx} = \left[\dfrac{\cos x + \sin x}{3x^2} \right]$

$\dfrac{d^2y}{dx^2} = \dfrac{-\sin x + \cos x}{6x}$

$\dfrac{d^2y}{dx^3} = \dfrac{-\cos x - \sin x}{6}$

$\lim_{x \to 0} = \dfrac{-\cos 0 - \cos 0}{6} = \dfrac{-1 - 0}{6} = -\dfrac{1}{6}$

$\lim \left[\dfrac{\sin x - \cos x}{} \right] = -1$

2A

a.) $U_n = \frac{2}{(n+1)(n+2)}$

$U_{n+1} = \frac{2}{(n+2)(n+3)}$

ratio : $\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{2}$

$$\frac{U_{n+1}}{U_n} = \frac{n+1}{n+3}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{n+1}{n+3}$$

$$= \frac{\frac{n}{n}+\frac{1}{n}}{\frac{n}{n}+3/n} = \frac{1+1/n}{1+3/n} = \frac{1+0}{1+0} = \frac{1}{1} = 1$$

Since $\lim_{n\to\infty} \frac{U_{n+1}}{U_n}$

∴ The Series is inconclusive

2B

(b.) Using the comparison test

recall

$$\left[\frac{1}{1^r}+\frac{1}{2^r}+\frac{1}{3^r}+\frac{1}{4^r}+\dots\frac{1}{n^r}\right] = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

$$\Rightarrow \left[\frac{2}{1^2}+\frac{2}{2^2}+\frac{2}{3^2}+\frac{2}{4^2}+\dots\frac{2}{n^2}\right] = \sum_{n=1} \frac{2}{n^2}$$

~~Since~~ $P = 2$

Since $P > 1$, The Series Converge

1b

1b.) $\lim_{x \to \pi/2} \ln\left(\frac{\exp(3x^2+2x-1)}{x+1}\right)$

$= \lim_{x \to \pi/2} \ln\left(\exp\left[\frac{(3x-1)(x+1)}{x+1^2}\right]\right)$

$= \lim_{x \to \pi/2} \ln(\exp(3x-1))$

$= \lim_{x \to \frac{\pi}{2}} (3x-1) \quad = 3\left(\frac{\pi}{2}\right) - 1$

$= \frac{3}{2}\pi - 1 \quad = \frac{3\pi}{2} - \frac{1}{1}$

$\therefore \lim \ln\left[\exp \frac{(3x^2+2x-1)}{x+1}\right] = \frac{3\pi-2}{2}$ //

1c

1c. $\lim_{x \to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right)$

$= \lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$= \cos\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \cos(\sin^{-1}(0.8660))$

$= \cos 60^\circ$

$= 1/2$ //

1D

d.) $\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$= \lim \left[(x-4)(x-4)\right]$

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1a

1(a) $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right]$

$= \left[\frac{\left(\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right) \sin(\cos \pi/2)}{\pi/2 - \pi/2} \right]$ $= \frac{\left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \sin\left[\cos \frac{\pi}{2}\right)}{0}$

∴ Using l'hopital's rule, ~~dy/dx of the~~ $U \, du/dx + V \, du/dx$

$\frac{dy}{dx}$ = Let $U = x^2 - \frac{\pi}{4}$

and $V = \sin(\cos x)$

$\frac{dy}{dx} = 2x$ $\quad \frac{dy}{dx} = ?$

$\frac{d}{dx} = \sin(\cos x)$ = Let $\cos x = w$

$V = \sin w$

$\frac{dv}{dw} = \cos w$, ∴ $\frac{dw}{dx} = -\sin x$

$\frac{dv}{dx} = \frac{dv}{dw} \times \frac{dw}{dx} = -\sin x \cos(\cos x)$

$= \frac{\left(x^2 - \frac{\pi}{4}\right) \times -\sin x \cos(\cos x) + \sin(\cos x)(2x)}{1}$

$= \left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4} \times -\sin 90 \cos(\cos 90) + \sin \cos 90 \times 2\left(\frac{\pi}{2}\right)$

$= \left(\frac{\pi^2}{4} - \frac{\pi}{4}\right) \times -1 + 0 \times \pi$

$= -\pi^2 + \pi \quad = \pi - \pi^2$