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Q) Evaluate

$$\lim_{x\to \pi/2}\left[\frac{\left(x^2-\pi/4\right)\left(\sin \cos x\right)}{x-\pi/2}\right]$$

using product rule $v\frac{du}{dx} + u\frac{dv}{dx}$ $\quad v = x^2-\pi/4$, $U = \sin(\cos x)$

$\frac{dv}{dx} \Rightarrow f = \cos x$ $\quad U = \sin p$

$\frac{df}{dx} = -\sin x$ $\quad \frac{du}{dx} = \cos p$

$\frac{du}{dx} = -\sin x \cos x(\cos x)$

$V = x^2 - \pi/4$, $\frac{dv}{dx} = 2x$

$$\frac{(x^2-\pi/4)(-\sin x \cos(\cos x)) + \sin(\cos x)(2x)}{1}$$

$\lim_{x\to\pi/2} ((\pi/2)^2-\pi/4)-(-\sin\pi/2)\cos(\cos\pi/2) + \sin(\cos\pi/2)(2x\pi/2)$

$= (\pi^2/4 - \pi/4)(-1\cos(0))$

$= \frac{\pi^2}{4} - \pi/4 \times -1$

$= \frac{-\pi^2}{4} + \frac{\pi}{4}$

$$\Rightarrow \lim_{x\to\pi/2}\left[\frac{(x^2-\pi/4)(\sin(\cos x))}{x-\pi/2}\right] = -\frac{\pi^2}{4} + \frac{\pi}{4}$$

b) $\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$

$\Rightarrow$ factorise: $(3x^2+2x-1)$

$\Rightarrow (3x-1)(x+1)$

$\Rightarrow \ln\left[\exp\frac{(3x-1)(x+1)}{x+1}\right]$

$\ln\left[\exp(3x-1)\right]$

$\lim_{x \to \pi/2} \Rightarrow \ln\left[\exp\left(\frac{3\pi}{2}-1\right)\right]$

$\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right] = \frac{3\pi}{2} - 1$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right]$

$\Rightarrow \cos\left[\frac{\sin^{-1}(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$\cos\left[\sin^{-1}\left(\sqrt{3}/2\right)\right]$

$\cos\left[60^\circ\right]$

$= 1/2$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$

d) $\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right]$

$\Rightarrow$ factorize:

$\lim_{x \to 4}\left[\frac{(x-4)(x-4)}{(x-1)(x-4)}\right]$

$\lim_{x \to 4}\left[\frac{x-4}{x-1}\right]$

$\Rightarrow \frac{4-4}{4-1}$

$\lim_{x \to 4}\left[\frac{x^2-8x+16}{x^2-5x+4}\right] = 0/3 = 0$

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2) a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6}$

Compare to Standard Converging Series

$\Rightarrow \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \cancel{\frac{1}{7^p}} + \cdots$

When $P = 2$

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots$

$\frac{2}{1\times3} < \frac{1}{1^2}$, $\frac{1}{2\times3} < \frac{1}{2^2}$, $\frac{2}{4\times5} < \frac{1}{3^2}$, $\frac{2}{5\times6} < \frac{1}{4^2}$

Since each term is less than its Corresponding term in the Series ;

$\therefore$ The series CONVERGES

b) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

$\Rightarrow U_n = \frac{2}{n^2}$ $\quad U_{n+1} = \frac{2}{(n+1)^2}$, $U_{n+1} = \frac{2}{n^2+2n+1}$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n^2+2n+1}{n^2}$$

$$= \frac{n^2/n^2 + 2n/n^2 + 1/n^2}{n^2/n^2}$$

$$= \frac{1 + (2\times0)/0 + 1/0}{1}$$

$$= \frac{1+0+0}{1} = 1$$

from the rule the Series may convergent or divergent

from test 1 $\lim_{n\to\infty} U_n = \lim_{n\to\infty} 2/n^2$

$= \cancel{4/n^2}$

$= \frac{2/n^2}{n^2/n^2} = 0/1 = 0$

$\therefore$ This Series CONVERGES