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1 Evaluate the following limits of functions

a $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4}) \sin(\cos x)}{x - \frac{\pi}{2}} \right]$

Soln

Applying product rule, $U\frac{dV}{dx} + V\frac{dU}{dx}$

for numerator

$U = x^2 - \pi/4$, $V = \sin(\cos x)$

$b = \cos x$, $V = \sin b$

$\frac{db}{dx} = -\sin x$, $\frac{dV}{db} = \cos b = \overset{\text{Recall}}{\cos(\cos x)}$

$\frac{dV}{dx} = \frac{dV}{db} \times \frac{db}{dx} = \sin x \times \cos(\cos x)$

$\frac{dy}{dx} = -\sin x \cos(\cos x)$

$\therefore \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(-\sin x \cos x) + \sin(\cos x)(2x)}{x - \frac{\pi}{2}} \right]$

denominator : $\frac{dy}{dx}\{x - \frac{\pi}{2}\} = 1$

$\lim_{x \to \pi/2} = \left[\{(\frac{\pi}{2})^2 - \frac{\pi}{4}\} \cos(\cos\frac{\pi}{2}) . \sin x + 2(\pi/2) \sin(\cos\frac{\pi}{2}) \right]$

$= \pi \sin 0 + \{\frac{\pi^2}{4} - \frac{\pi}{4}\} \overset{1}{\cos 0} (-\sin\frac{\pi}{2})$

$= 0 + \{\frac{\pi^2}{4} - \frac{\pi}{4}\} \overset{1}{1} . (-1)$

$\lim_{x\to\frac{\pi}{2}} \pi = \left[\frac{\pi^2}{4} + \frac{\pi}{4}\right] = -\frac{\pi^2}{4} + \frac{\pi}{4}$

1b $\lim_{x\to\frac{\pi}{2}} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right]$

Soln

factorising $(3x^2+2x-1)$

$3x^2+3x-x-1 = 3x(x+1)-1(x+1)$

$= (3x-1)(x+1)$

$\exp\frac{(3x^2+2x-1)}{x+1} = \exp\frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$

$= \exp(3x-1)$

$\lim_{x\to\pi/2} \ln[\exp(3x-1)$

$= \lim_{x\to\pi/2} \ln[\exp 3(\pi/2)-1]$

$= \lim_{x\to\pi/2} \ln[\exp 3(\pi/2)-1] = \ln(40.9515)$

$\therefore \lim_{x\to\frac{\pi}{2}} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right) = 3.7124$ //

1c $\lim_{x\to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right)$

Soln Solving directly

$\left[\sin^{-1}\left\{\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right\}\right]$

$= \left[\sin^{-1}\frac{(\sqrt{3})}{2}\right] = 60$

$\cos(60) = 1/2$

$\therefore \lim_{x\to 2+\sqrt{3}} \cos\left(\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right) = 1/2$

d $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$

Soln

Solving directly

$$\lim_{x \to 4} \left[\frac{(4)^2 - 8(4) + 16}{(4)^2 - 5(4) + 4}\right]$$

$$= \left[\frac{16 - 32 + 16}{16 - 20 + 4}\right] = \frac{0}{0} \quad \text{indeterminate}$$

Applying l'Hopital's rule

$$\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{2x^2 - 5x + 4}\right] = \lim_{x \to 4} \left[\frac{2x - 8}{2x - 5}\right] = \frac{2(4) - 8}{2(4) - 5}$$

$$= \frac{8 - 8}{8 - 5} = \frac{0}{3}$$

The result is undefined.

2 Determine whether each of the following series is convergent

a $\frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \frac{2}{5 \times 6} + \cdots$

Soln

Comparing the series to a standard series that converges

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \cdots$$

When $p = 2$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6} + \cdots$$

$$\frac{2}{1\times3} < \frac{1}{1^2}, \quad \frac{1}{2\times3} < \frac{1}{2^2}, \quad \frac{2}{4\times5} < \frac{1}{3^2}, \quad \frac{2}{5\times6} < \frac{1}{4^2}$$

Each term in the series is less than the corresponding term in the series known to converge. Therefore the series is convergent.

1b

2b $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

i. Soln

Comparing the series to a standard series that converge

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \dots$$

when $p = 2$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots$$

$$\frac{2}{1^2} > \frac{1}{1^2}, \quad \frac{2}{2^2} > \frac{1}{2^2}, \quad \frac{3}{3^2} > \frac{1}{3^2}, \quad \frac{2}{4^2} > \frac{1}{4^2}$$

Each term of the series is greater than the corresponding term in the series, Therefore, the given series diverges.

2c $U_n = \dfrac{1+2n^2}{1+n^2}$

Soln

$$U_n = \frac{1+2n^2}{1+n^2}, \quad U_{n+1} = \frac{1+2(n+1)^2}{1+(n+1)^2}$$

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$$\frac{U_{n+1}}{U_n} = \frac{1+2(n+1)^2}{1+(n+1)^2} \times \frac{1+n^2}{1+2n^2}$$

take where $n = n+1$

$=$

$= \cancel{1+2n^2+4}$

take $(n+1)^2 = n^2+2n+1$

$$\frac{U_{n+1}}{U_n} = \frac{1+2(n^2+2n+1)}{1+n^2+2n+1} \times \frac{1+n^2}{1+2n^2}$$

$$= \frac{(1+2n^2+4n+3)(1+n^2)}{(1+n^2+2n+2)(1+2n^2)}$$

$$= \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^2}$$

$$= \frac{\cancel{5n^2}\ 2n^4+4n^3+5n^2+4n+3}{2n^4+4n^3+5n^2+2n+2}$$

$$\lim_{n\to\infty} = \left\{ \frac{2+\frac{4}{n}+\frac{5}{n^2}+\frac{4}{n^3}+\frac{3}{n^4}}{2+\frac{4}{n}+\frac{5}{n^2}+\frac{2}{n^3}+\frac{2}{n^4}} \right\}$$

$$= \frac{2}{2} = 1 \quad (\text{indeterminable})$$

3 Find the range of values ~~for~~ of x for which the series below is absolute convergence

$$\frac{x}{27} + \frac{x^2}{125} + \ldots + \frac{x^n}{(2n+1)}$$

Soln

Soln

$$|U_n| = \frac{x^n}{(2n+1)^3}$$

$$|U_{n+1}| = \frac{x^{n+1}}{(2n+1+1)^3} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\text{Ratio } \frac{|U_{n+1}|}{|U_n|} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(2n+1)^3}{(2n+2)^3}$$

$$\lim_{n \to \infty} \frac{|U_{n+1}|}{|U_n|} = \lim_{n \to \infty} \left\{ \frac{x(2+\frac{1}{n})^3}{1(2+\frac{2}{n})^3} \right\}$$

$$= \frac{x}{1} \frac{(2)^3}{(2)^3} = x$$

$\therefore \quad -1 < x \leq 1$ is the range of value of x

4 Evaluating using L'Hopital's Rule.

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

Soln

Solving directly gives $\frac{0-1}{(0)^3} = 0$ (indeterminate)

Applying L'Hopital's Rule

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right] = \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right] = \frac{1-0}{3(0)^2}$$

$$= 0$$

Apply the rule 2^{nd} time

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3}\right] = \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2}\right] =$$

$$\lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x}\right] = \frac{0+1}{6(0)} = 0$$

Applying the rule 3^{rd} time

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3 \cancel{\sin}}\right] = \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2}\right] =$$

$$\lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x}\right] = \lim_{x \to 0} \left[\frac{-\cos x - \sin x}{6}\right] = \frac{-1-0}{6}$$

$$= \frac{-1}{6} //$$