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COMPUTER ENGINEERING
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ENG 281 assignment 2.

The parametic equation of a curve are as given on equation (i) & (ii)

$x = \cos t + t\sin t$ — (i)

$y = \sin t - t\cos t$ — (ii).

(i) An expression for the radius (R) and
(ii) Expression for the co-ordinates of the circle of curvature

Answer

a) $x = \cos t + t\sin t$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$

$y = \sin t - t\cos t$

$\frac{dy}{dt} = \cos t + t\sin t - \cos t - t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{t\sin t}{t\cos t}$

$= \frac{dy}{dx} = \frac{\sin t}{\cos t}$

$= \frac{d^2y}{dx^2} \Rightarrow u = \sin t, \quad v = \cos t$

$\frac{du}{dt} = \cos t, \quad \frac{dv}{dt} = -\sin t$

$= \frac{v\frac{dy}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{\cos t(\cos t) - (\sin t)(-\sin t)}{(\cos t)^2} \times \frac{1}{t\cos t}$

$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{1}{t\cos t}$

$$\Rightarrow \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

$$= \frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

$$= R = \frac{\left[1 + \left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left[1 + \left[\frac{\sin t}{\cos t}\right]^2\right]^{3/2}}{d^2y/dx^2}$$

$$R = \frac{\left[1 + \left[\frac{\sin^2 t}{\cos^2 t}\right]^2\right]^{3/2}}{1/t\cos^3 t}$$

$$R = \left[\frac{\cos^2 t + \sin^2 t}{\cos^2 t}\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$= R = \left[1/\cos^2 t\right]^{3/2} \div \frac{1}{t\cos^3 t}$$

$$R = \frac{1/3}{\frac{1}{t}\cos^3 t} \cos^3 t \qquad = R = \frac{1}{\cos^3 t} \times t\cos^3 t$$

$$\therefore R = t$$

b) $h = x_0 = R\sin t$

$k = y_1 + R\cos\theta$

$R = t \quad , \quad \theta = \tan^{-1}\left[dy/dx\right]$

$$\frac{dy}{dx} = \frac{\sin t}{\cos t}$$

$$= \tan t$$

$\therefore \theta = \frac{1}{t \cos\theta} \times t \cos t$

$\theta = t$

$\therefore h = x_0 - t \sin t$

$k = y_0 + t \cos t$

but $x_0 = \cos t + t \sin t$

$y_0 = \sin t - t \cos t$

$h = \cos t + t \sin t - t \sin t$

$h = \cos t$

$k = \sin t - t \cos t + t \cos t$

$k = \sin t$

$[h, k] = [\cos t, \sin t]$