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1a) Evaluate: $\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4}) \cdot (\sin(\cos x))}{x - \frac{\pi}{2}} \right]$

Solution

Applying Product rule; $U\frac{dv}{dx} + V\frac{du}{dx}$
for the numerator

$U = x^2 - \frac{\pi}{4}$, $V = \sin(\cos x)$

$a = \cos x \Rightarrow V = \sin a$

$\frac{da}{dx} = -\sin x$ $\quad \frac{dv}{dx} = \cos a = \cos(\cos x)$

$\frac{dv}{da} \times \frac{da}{dx} = \frac{dv}{dx} = \sin x \cdot \cos(\cos x)$

$\frac{dv}{dx} = -\sin x \cdot \cos(\cos x)$

$\therefore$

$$\lim_{x \to \frac{\pi}{2}} \left[\frac{(x^2 - \frac{\pi}{4})(-\sin x \cdot \cos x) + \sin(\cos x)(2x)}{x - \frac{\pi}{2}} \right]$$

$$= \frac{\left[\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right] \cos(\cos\frac{\pi}{2}) \cdot \sin x + 2\left(\frac{\pi}{2}\right) \sin(\cos\frac{\pi}{2})}{1}$$

$$= \frac{\pi \sin 0 + \left[\frac{\pi^2}{4} - \frac{\pi}{4}\right] \cos 0 \cdot - \sin\frac{\pi}{2}}{1}$$

$$= 0 + \left[\frac{\pi^2}{4} - \frac{\pi}{4}\right] 1 \cdot -1$$

$$= \frac{-\pi^2}{4} + \frac{\pi}{4}$$

1b Evaluate $\lim \ln \left[\exp \dfrac{(3x^2+2x-1)}{x+1}\right]$

Solution

Factorizing $(3x^2+2x-1)$

$$3x^2+3x-x-1 = 3x(x+1)-1(x+1)$$
$$= (3x-1)(x+1)$$

$$\exp\frac{(3x^2+2x-1)}{x+1} = \exp\frac{(3x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$
$$= \exp(3x-1)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \ln \left[\exp(3x-1)\right]$$
$$= \lim_{x \to \frac{\pi}{2}} \ln \left[\exp(3(\tfrac{\pi}{2})-1)\right]$$
$$= \ln(40.9515)$$
$$= \underline{3.7124}$$

1c $\lim \cos \left[\dfrac{\sin^{-1}(x-2)}{x-\sqrt{3}}\right]$

Solution

$$\left[\frac{\sin^{-1}(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$$
$$= \left[\frac{\sin^{-1}(\sqrt{3})}{(2)}\right] = 60$$

$$\cos(60) = \tfrac{1}{2}$$

$$\lim \cos \left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right] = \frac{1}{2}$$

d $\lim_{x \to 4} \left[\dfrac{x^2-8x+16}{x^2-5x+4}\right]$: Evaluate

Solution

$$= \frac{[(x-4)\cancel{(x-4)}]}{(x-1)\cancel{(x-4)}}$$

$$\lim_{x \to 4} \left[\frac{(x-4)}{(x-1)}\right] = \frac{4-4}{4-1} = \frac{0}{3} = 0$$

2a) $\dfrac{2}{2\times 3} + \dfrac{2}{3\times 4} + \dfrac{2}{4\times 5} + \dfrac{2}{5\times 6} + \dots$

Comparing the series to a standard series that converges

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \dots$$

When $P = 2$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots$$

$$\frac{2}{1\times 3} < \frac{1}{1^2}, \quad \frac{1}{2\times 3} < \frac{1}{2^2}, \quad \frac{2}{4\times 5} < \frac{1}{3^2}, \quad \frac{2}{5\times 6} < \frac{1}{4^2}$$

Each term of the given series is less than the corresponding term in the series known to converge, Therefore the given series converges.

2b) $\dfrac{2}{1^2} + \dfrac{2}{2^2} + \dfrac{2}{3^2} + \dfrac{2}{4^2} + \dots$

Comparing the series to a standard series that converges.

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \dots$$

When $P = 2$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots$$

$$\frac{2}{1^2} > \frac{1}{1^2}, \quad \frac{2}{2^2} > \frac{1}{2^2}, \quad \frac{2}{3^2} > \frac{1}{3^2}, \quad \frac{2}{4^2} > \frac{1}{4^2}$$

b) Each term of the given series is greater than the corresponding term in the series known to converge. Therefore, the given series diverges.

2c) $U_n = \frac{1+2n^2}{1+n^2}$

$U_{n+1} = \frac{1+2(n+1)^2}{(n+1)^2+1} \times \frac{n^2+1}{1+2n^2}$

$\frac{U_{n+1}}{U_n} = \frac{1+2n^2}{1+n^2} = \frac{1+2(n+1)^2}{1+2n^2}$

$= \frac{1+2n^2}{1+n^2} = \frac{1+2(n+1)^2}{1+(n+1)^2}$ $(n=n+1)$

$= \frac{1+2n^2+4n+2}{1+n^2+2n+1} \times \frac{1+n^2}{1+2n^2}$

$= \frac{(2n^2+4n+3)(1+n^2)}{(n^2+2n+2)(1+2n^2)}$

$= \frac{2n^2+4n+3+2n^4+4n^3+3n^2}{n^2+2n+2+2n^4+4n^3+4n^2}$

$= \frac{5n^2+2n^4+4n^3+4n+3}{5n^2+2n^4+4n^3+2n+2}$

$\lim_{x \to \infty} = \frac{\left\{ \frac{5}{n^2}+2+\frac{4}{n}+\frac{4}{n^3}+\frac{3}{n^4} \right\}}{\left(\frac{5}{n^2}+2+\frac{4}{n}+\frac{2}{n^3}+\frac{2}{n^4} \right)}$

$= \frac{2}{3} = 1$ (Inconclusive)

3) $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{nx^n}{(2n+1)^3}$

$U_n = \frac{x^n}{(2n+1)^3}$

$U_{n+1} = \frac{x^{n+1}}{[2(n+1)+1]^3} = \frac{x^{n+1}}{(2n+3)^3}$

$\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x_n} = \frac{x^n \cdot x^1}{(2n+3)^3}$

$\frac{U_{n+1}}{U_n} = \frac{x(2n+1)^3}{(2n+3)^3} = \frac{x(8n^3+12n^2+6n+1)}{(8n^3+36n^2+54n+27)}$

$\frac{U_{n+1}}{U_n} = x \frac{\left(8+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right)}{8+\frac{36}{n}+\frac{54}{n^2}+\frac{27}{n}}$

$n \to 0 = x\frac{(8)}{8} = x$

$U_\infty = \frac{U_{n+1}}{U_n} = x$

$\Rightarrow -1 < x \leq 1$ is the range of values of x.

4) $\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

sub $x \Rightarrow 0$

$= \left(\frac{\sin 0 - \cos 0}{0} \right)$

$= \frac{0}{0}$ = undefined

$\frac{d}{dx}\left(\frac{\sin x - \cos x}{x^3} \right) = \frac{\cos x + \sin x}{3x^2}$

sub $x = 0$

$= \frac{\cos 0 + \sin 0}{3(0)} = \frac{1+0}{0}$ = undefined

$\frac{d}{dx}\left(\frac{\cos x + \sin x}{3x^2} \right) = \frac{-\sin x + \cos x}{6x}$

$\frac{-\sin(0) + \cos(0)}{6(0)} = \frac{0+1}{0}$ = undefined

$\frac{d}{dx}\left(\frac{-\sin x + \cos x}{6x} \right) = \frac{-\cos x - \sin x}{6}$

Sub $x = 0$

$$\frac{-\cos(0) - \sin(0)}{6} = \frac{-1-0}{6} = \frac{-1}{6}$$