NAME: ALA-BINTE TARILATE EDWIN
MAT NO: 15/ENG07/005

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DEPT: PETROLEUM ENGINEERING

ENG 381 ASSIGNMENT

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Soln

$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$y'' - y' - 2y = 8$

$k^2 - k - 2 = 8$

$k^2 - k - 2 = 0$

$(k-2)(k+1) = 0$

$k_1 = 2$, $k_2 = -1$

$y_h = C_1y_1 + C_2y_2$

$y = C_1e^{2x} + C_2e^{-x}$

$y_p = A$

$y_p' = 0$

$y_p'' = 0$

$-2A = 8$

$A = -4.$

The general solution becomes

$y = y_h + y_p$

$y = C_1e^{2x} + C_2e^{-x} - 4.$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Soln

$\frac{d^2y}{dx^2} = y''$, $y = e^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} - 4e^{kx} = 10e^{3x}$

$(k^2 - 4)e^{kx} = 10e^{3x}$

$4e^{2x}$

$k^2 - 4 = 0$

$k = \pm 2$

$y_h = C_1y_1 + C_2y_2$

$C_1e^{2x} + C_2e^{-2x}$

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Considering the particular part of the eqn y_p

let $y_p = Ae^{3x}$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

$\therefore\ 9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$5Ae^{3x} = 10e^{3x}$

$5A = 10$

$A = 2$

$y_p = 2e^{3x}$

$\therefore$ The general solution becomes

$y = C_1e^{2x} + C_2e^{-2x} + 2e^{5x}$

8) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Soln

$d^2y/dx^2 = y''$, $dy/dx = y'$, $y = e^{kx}$

$k^2e^{kx} + 2ke^{kx} + e^{kx} = e^{-2x}$

$(k^2 + 2k + 1)e^{kx} = e^{-2x}$

Assume Homogeneity

$k^2 + 2k + 1 = 0$

$k^2 + k + k + 1 = 0$

$(k+1)(k+1) = 0$

$k = -1$

$y = C_1y_1 + xC_2y_2$

$y = C_1e^{-x} + xC_2e^{-x}$

Considering the particular part

$y_p = Ae^{-2x}$

$y_p' = -2Ae^{-2x}$

$y_p'' = 4Ae^{-2x}$

$\therefore\ 4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$\therefore\ y = C_1e^{-x} + xC_2e^{-x} + e^{-2x}$, $y = e^{-x}(C_1 + xC_2) + e^{-2x}$

4) $\frac{d^2y}{dx^2} + 2$

Soln

$\frac{d^2y}{dx^2} = y$

$k^2e^{kx} +$

Assume

$(k^2+2$

$k = \pm$

Since we

$y_h = A$

let $y_p =$

Substit

$2A +$

$2A + 2$

$25A$

Compari

$25A =$

$25B =$

$2A + 2$

$2C/$

5) $\frac{d^2y}{dx^2}$

Assume

$k^2 -$

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4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Soln

$\frac{d^2y}{dx^2} = y''$, $y = e^{kx}$, $y'' = ke^{kx}$

$k^2e^{kx} + 25e^{kx} = 5x^2 + x$

Assume homogeneity

$(k^2 + 25) = 0$

$k = \pm 5i$

Since we have complex roots

$y_h = A\cos 5x + B\sin 5x$

Let $y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

Substituting we have

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$25Ax^2 + 25Bx + 2A + 25C = 5x^2 + x$

Comparing Coefficients

$25A = 5$; $A = 1/5$

$25B = 1$; $B = 1/25$

$2A + 25C = 0$

$2(1/5) + 25C = 0$; $C = -2/125$

$y_p = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = y_h + y_p$

$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Soln

Assume homogeneity

$k^2 - 2k + 1 = 0$

$(k-1)(k-1) = 0$

$k = 1$

$y_h = (C_1 + xC_2)e^x$

$y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$, $y'' = -A\sin x - B\cos x$

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$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x = 4\sin x$

$+2B\sin x - 2A\cos x = 4\sin x + 0\cos x$

Comparing Coefficients

$2B = 4 ; B = 2$

$-2A = 0 ; A = 0$

$A = 0, B = 2$

$y_p = 2\cos x$

$y = y_h + y_p$

$y = e^{x}(C_1 + C_2 x) + 2\cos x$

6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that $x = 0, y = 1$ and $dy/dx = -2$

Soln

Assume homogeneity

$k^2 + 4k + 5 = 0$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$k_1 = -2 + i ; k_2 = -2 - i$

$y_h = e^{-2x}(C_1\cos x + C_2\sin x)$

let $y_p = Ae^{-2x}$

$y_p' = -2Ae^{-2x}$

$y_p'' = 4Ae^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5Ae^{-2x} = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$A = 2$

$y = e^{-2x}(C_1\cos x + C_2\sin x) + 2e^{-2x}$

$y' = e^{-2x}(-C_1\sin x + C_2\cos x) + (-2e^{2x})(C_1\cos x + C_2\sin x) - 4e^{-2x}$

when $x = 0, y = 1, y' = -2$

$y = 1 = e^{-2(0)}(C_1\cos(0) + C_2\sin(0)) + 2e^{-2(0)}, \; 1 = C_1 + 2, \; C_1 = -1$

$y' = -2 = e^{-2(0)}(-C_1\sin(0) + C_2\cos(0)) + (-2e^{2(0)})(C_1\cos 0 + C_2\sin(0))$

$-2 = C_2 - 2C_1 - 4, \; C_2 - 2C_1 = 2, \; C_2 = 0$

$y = e^{-2x}(-\cos x + 0) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

$y = e^{-2x}(2 - \cos x)$

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4) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x-3$

Soln

Assume homogeneity

$3k^2-2k-1=0$

$(3k+1)(k-1)=0$

$k_1=1,\ k_2=-\frac{1}{3}$

$y_h = C_1e^{x} + C_2e^{-x/3}$

Considering the particular part

Let $y_p = Ax - B$

$y_p' = A$

$y_p'' = 0$

$3(0) - 2(A) - (Ax - B) = 2x - 3$

$-2A - Ax + B = 2x - 3$

$-A = 2;\ A = -2$

$-2A + B = -3$

$-2(-2) + B = -3$

$B = -7$

$y_p = -2x - (-7)$

$y_p = -2x + 7$

∴ The general solution y is given as

$y = y_h + y_p$

$y = C_1e^{x} + C_2e^{-x/3} - 2x + 7$

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8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln

Assuming homogenity

$k^2 - 6k + 8 = 0$

$k^2 - 4k - 2k + 8 = 0$

$K(K-4) - 2(k-4) = 0$

$(k-4)(k-2) = 0$

$k_1 = 4,\ k_2 = 2$

$y_h = C_1e^{2x} + C_2e^{4x}$

Considering the particular Part

Let $y_p = Axe^{4x}$

$y_p' = x4Ae^{4x} + Ae^{4x}$

$y_p'' = 4A(4e^{4x}\cdot x + e^{4x}) + 4Ae^{4x}$

$y_p'' = 16Axe^{4x} + 8Ae^{4x}$

$16Axe^{4x} + 8Ae^{4x} - 6(Axe^{4x} + Ae^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

$16Axe^{4x} + 8Ae^{4x} - 24Axe^{4x} - 6Ae^{4x} + 8Axe^{4x} = 8e^{4x}$

$2Ae^{4x} = 8e^{4x}$

$2A = 8$

$A = 4.$

$y_p = 4xe^{4x}$

The general solution

$y = y_h + y_p$

$y = C_1e^{2x} + C_2e^{2x} + 4xe^{4x}$