

mechanical Eng
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10) The parametric equation of a curve are as given in equation 1 & 2

$$x = \cos t + \sin t - 1$$

$$y = \sin t - t \cos t - 2$$

in terms of t determine

20) An expression for the radius of curvature

Solution

$$R \Rightarrow \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} \frac{dx}{d^2y}$$

10) $x = \cos t + \sin t$
 $\frac{dx}{dt} = -\sin t + \cos t$

$$\frac{dx}{dt} = \cos t$$

$$y = \sin t - t \cos t$$

$$\frac{dy}{dt} = \cos t - (-t \sin t + \cos t)$$

$$= \cos t + t \sin t - \cos t$$

$$= t \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= t \sin t \times \frac{1}{\cos t} = \frac{t \sin t}{\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left[\frac{dy}{dx} \right] = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{dt}{dx}$$

$$= \frac{d}{dt} \left(\frac{\sin t}{\cos t} \right) \times \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = U \frac{dy}{dx} - U \frac{du}{dx} \quad \text{given}$$

where $U = \cos t$
 $du = \sin t$
 $u = \sin t$

$$\frac{d^2 y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

Recall that $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{\cos t} = \frac{1}{\cos^3 t}$$

Also recall that

$$R = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}$$

$$R = \left[1 + \left(\frac{\sin t}{\cos t} \right)^2 \right]^{3/2}$$

$$R = \left[1 + \frac{\sin^2 t}{\cos^2 t} \right]^{3/2} \times \frac{t \cos^3 t}{1}$$

$$R = \left(\frac{\cos^2 t + \sin^2 t}{\cos^2 t} \right)^{3/2} \times t \cos^3 t$$

$$R = \frac{1}{(\cos^2 t)^{3/2}} \times t \cos^3 t = \frac{1}{(\sqrt{\cos^2 t})^3} \times t \cos^3 t$$

$$R = \frac{t \cos^3 t}{\cos^3 t} = t$$

\therefore The radius of curvature is t

Expressions for the co-ordinates (h, k) of the centre of curvature

$$\Rightarrow h = x_1 - R \sin \theta - 1$$

$$k = y_1 + R \cos \theta - 2$$

$$R = t \quad t = \theta$$

$$x_1 = \cos t + t \sin t$$

$$y_1 = \sin t + t \cos t$$

Substituting x_1 in eqn 1 and y_1 in eqn 2

$h = \cos t$
 $h = \sin t$
 $k = \sin t$
 $k = \sin t$
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 centre of

$$h = \cos t + t \sin t - t \sin t$$

$$h = \cos t$$

$$k = \sin t - t \cos t + t \cos t$$

$$k = \sin t$$

The expressions for the coordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$.