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ASSIGNMENT I

(1.) $\frac{d^2y}{dx^2} - dy/dx - 2y = 8$

$y'' - y' - 2y = 8$

$k^2 - k - 2 = 0$

$(k^2 - 2k)(+k - 2) = 0$

$k(k-2) + 1(k-2) = 0$

$(k-2)(k+1) = 0$

$k_1 = 2$ and $k_2 = -1$

$y = c_1 e^{2x} + c_2 e^{-x}$

$y_p = A$

$y'_p = 0$

$y''_p = 0$

$0 - 0 - 2A = 8$

$-2A = 8$

$A = -8/2$

$A = -4$

$\therefore y_p = -4$

$y_c = y_h + y_p$

$y = c_1 e^{2x} + c_2 e^{-x} - 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{2x}$

$y'' - 4y = 10e^{3x}$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \pm\sqrt{4}$

$k = 2$ or -2

$k_1 = 2$ and $k_2 = -2$

$y_h = c_1 e^{2x} + c_2 e^{-2x}$

$y_p = Ae^{3x}$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

$9Ae^{3x} - 4(Ae^{3x}) = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$9A - 4A = 10$

$+5A = 10$

$A = +10/5$

$y_p = \frac{+10}{5} e^{3x} = 2e^{3x}$

$y = y_h + y_p$

$y = c_1 e^{2x} + c_2 e^{-2x} + 2e^{3x}$ //

(3) $d^2y/dx^2 + 2dy/dx + y = e^{-2x}$

$y'' + 2y' + y = e^{-2x}$

$k^2 + 2k + 1 = 0$

$(k^2 + k)(+k + 1) = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$k_1 = -1$ and $k_2 = -1$

$y_h = c_1 y_1 + x c_2 y_2$

$y_h = c_1 e^{-x} + x c_2 e^{-x}$

$y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y_p'' = 4Ae^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$\cancel{4Ae^{-2x}} - \cancel{4Ae^{-2x}} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$\therefore y_p = e^{-2x}$

$y = y_h + y_p$

$y = C_1e^{-x} + xC_2e^{-x} + e^{-2x}$

$y = e^{-x}[C_1 + C_2x] + e^{-2x}$ //

(4) $d^2y/dx^2 + 25y = 5x^2 + x$

$y'' + 25y = 5x^2 + x$

$k^2 + 25 = 0$

$k^2 = -25$

$k = \pm\sqrt{-25}$

$k_1 = 5i$ and $k_2 = -5i$

$y_h = C_1e^{5xi} + C_2e^{-5xi}$

$y_h = A\cos 5x + B\sin 5x$

$y_p = Ax^2 + Bx + C$

$y'p = 2Ax + B$

$y''p = 2A$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$(25A)x^2 + (25B)x + (2A + 25C) = (5)x^2 + (1)x + 0$

$25A = 5$

$A = 5/25 = 1/5$

$25B = 1$

$B = 1/25$

$2A + 25C = 0$

$25C = -2A$

$25C = -2\left(\frac{1}{5}\right) = -\frac{2}{5}$

$C = -\frac{2}{5} \times \frac{1}{25} = -\frac{2}{125}$

$y_p = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$y = y_h + y_p$

$y = A\cos 5x + B\sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

(5) $d^2y/dx^2 - 2\,dy/dx + y = 4\sin x$

$y'' - 2y' + y = 4\sin x$

$k^2 - 2k + 1 = 0$

$(k^2 - k)(-k + 1) = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k_1 = 1$ and $k_2 = 1$

$y_h = C_1 y_1 + x C_2 y_2$

$y_h = C_1 e^x + x C_2 e^x$

$y_p = A\cos x + B\sin x$

$y'_p = -A\sin x + B\cos x$

$y''_p = -A\cos x - B\sin x$

$-A\cos x - B\sin x - 2\left[-A\sin x + B\cos x\right] + A\cos x + B\sin x = 4\sin x$

$-\cancel{A\cos x} - \cancel{B\sin x} + 2A\sin x - 2B\cos x + \cancel{A\cos x} + \cancel{B\sin x} = 4\sin x$

$(2A)\sin x + (-2B)\cos x = (4)\sin x + (0)\cos x$

$2A = 4$

$A = 4/2 = 2$

$-2B = 0$

$B = 0$

$y_p = 2\cos x + 0\sin x = 2\cos x$

$y = y_h + y_p$

$y = C_1 e^x + x C_2 e^x + 2\cos x$

$y = e^x\left[C_1 + C_2 x\right] + 2\cos x$

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(6) $d^2y/dx^2 + 4\,dy/dx + 5y = 2e^{-2x}$ given that at $x = 0$
$y = 1$ and $dy/dx = -2$

$y'' + 4y' + 5y = 2e^{-2x}$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + (2)^2 = -5 + (2)^2$

$(k+2)^2 = -5 + 4 = -1$

$k + 2 = \pm\sqrt{-1}$

$k = -2 \pm \sqrt{-1}$

$k_1 = -2 + i$ and $k_2 = -2 - i$

$y_h = c_1 e^{(-2+i)x} + c_2 e^{(-2-i)x}$

$y_h = c_1 e^{-2x + ix} + c_2 e^{-2x - ix}$

$y_h = c_1 e^{-2x}\cdot e^{ix} + c_2 e^{-2x}\cdot e^{-ix}$

$y_h = e^{-2x}[c_1 e^{ix} + c_2 e^{-ix}]$

$y_h = e^{-2x}[A\cos x + B\sin x]$

$y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5(Ae^{-2x}) = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$A = 2$

$y_p = 2e^{-2x}$

$y = y_h + y_p$

$y = e^{-2x}[A\cos x + B\sin x] + 2e^{-2x}$

$1 = e^{-2(0)}[A\cos(0) + B\sin(0)] + 2e^{-2(0)}$

$1 = A + 2$

$A = 1 - 2 = -1$

$y'' = \left[-2e^{-2x}[A\cos x + B\sin x]\right] + \left[e^{-2x}[-A\sin x + B\cos x]\right]$
$- 4e^{-2x}$

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$y'' = -2Ae^{-2x}\cos x - 2Be^{-2x}\sin x - Ae^{-2x}\sin x + Be^{-2x}\cos x$
$- 4e^{-2x}$

$-2 = -2Ae^{-2(0)}\cos(0) - 2Be^{-2(0)}\sin(0) - Ae^{-2(0)}\sin(0)$
$+ Be^{-2(0)}\cos(0) - 4e^{-2(0)}$

$-2 = -2A + B - 4$

$B = -2 + 2A + 4$

$B = -2 + 2(-1) + 4$

$B = -2 - 2 + 4$

$B = 0$

$y = e^{-2x}[-\cos x + 0\sin x] + 2e^{-2x}$

$y = -e^{-2x}\cos x + 2e^{-2x}$

$y = e^{-2x}[2 - \cos x]$

(7) $3\,d^2y/dx^2 - 2\,dy/dx - y = 2x - 3$

$3y'' - 2y' - y = 2x - 3$

$3k^2 - 2k - 1 = 0$

$(3k^2 - 3k)(+k - 1) = 0$

$3k(k - 1) + 1(k - 1) = 0$

$(k - 1)(3k + 1) = 0$

$k_1 = 1$ and $k_2 = -1/3$

$y_h = C_1e^{x} + C_2e^{-x/3}$

$y_p = Ax + B$

$y'_p = A$

$y''_p = 0$

$3(0) - 2(A) - (Ax + B) = 2x - 3$

$-2A - Ax - B = 2x - 3$

$(-A)x + (-2A - B) = (2)x + (-3)$

$-A = 2$

$A = -2$

$-2A - B = -3$

$-2(-2) - B = -3$

$B = 4 + 3 = 7$

$y_p = -2x + 7$

$y = y_h + y_p$

$y = C_1 e^{x} + C_2 e^{-x/3} - 2x + 7$

(8) $d^2y/dx^2 - 6\, dy/dx + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 8e^{4x}$

$k^2 - 6k + 8 = 0$

$(k^2 - 4k)(-2k + 8) = 0$

$k(k-4) - 2(k-4) = 0$

$(k-4)(k-2) = 0$

$k_1 = 4$ and $k_2 = 2$

$y_h = C_1 e^{4x} + C_2 e^{2x}$

$y_p = Axe^{4x}$

$y'_p = A[e^{4x} + 4xe^{4x}] = Ae^{4x} + 4Axe^{4x}$

$y''_p = 4Ae^{4x} + 4A[e^{4x} + 4xe^{4x}]$

$y''_p = 4Ae^{4x} + 4Ae^{4x} + 16Axe^{4x} = 8Ae^{4x} + 16Axe^{4x}$

$8Ae^{4x} + 16Axe^{4x} - 6(Ae^{4x} + 4Axe^{4x}) + 8(Axe^{4x}) = 8e^{4x}$

$8Ae^{4x} + \cancel{16Ax}e^{4x} - 6Ae^{4x} - \cancel{24Ax}e^{4x} + \cancel{8Ax}e^{4x} = 8e^{4x}$

$2Ae^{4x} = 8e^{4x}$

$2A = 8$

$A = 4$

$\therefore y_p = 4xe^{4x}$

$y = y_h + y_p$

$y = C_1 e^{4x} + C_2 e^{3x} + 4xe^{4x}$

ASSIGNMENT II

(12) $d^2y/d\theta^2 + 4\,dy/d\theta + 5y = 6\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + (2)^2 = -5 + (2)^2$

$(k+2)^2 = -1$

$k+2 = \pm\sqrt{-1}$

$k+2 = \pm i$

$k_1 = -2+i$ and $k_2 = -2-i$

$y_h = C_1 e^{(-2+i)\theta} + C_2 e^{(-2-i)\theta}$

$y_h = C_1 e^{-2\theta+i\theta} + C_2 e^{-2\theta-i\theta}$

$y_h = C_1 e^{-2\theta}\cdot e^{i\theta} + C_2 e^{-2\theta}\cdot e^{-i\theta}$

$y_h = e^{-2\theta}[C_1 e^{i\theta} + C_2 e^{-i\theta}]$

$y_h = e^{-2\theta}[A\cos\theta + B\sin\theta]$

$y_p = A\cos\theta + B\sin\theta$

$y'_p = -A\sin\theta + B\cos\theta$

$y''_p = -A\cos\theta - B\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta)$
$+ 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta$
$+ 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$4A\cos\theta + 4B\sin\theta - 4A\sin\theta + 4B\cos\theta$
$= 6\sin\theta$

$(-4A+4B)\sin\theta +$
$(4A+4B)\cos\theta = 6\sin\theta$

$-4A + 4B = 6$

$4A + 4B = 0$

$8B = 6$

$B = 6/8 = 3/4$

$4A = -4B$

$A = -B$

$A = -3/4$

$\therefore y_p = -3/4\cos\theta + 3/4\sin\theta$

$y = y_h + y_p$

$y = e^{-2\theta}[A\cos\theta + B\sin\theta]$
$+ 3/4\sin\theta - 3/4\cos\theta$

(ii) Steady state equation

$y_p' = 0$

$y_p' = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\therefore \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\frac{3}{4}\cos\theta = -\frac{3}{4}\sin\theta$

$\cos\theta = -\sin\theta$

$\frac{\sin\theta}{\cos\theta} = -\frac{\cos\theta}{\cos\theta}$

$\tan\theta = -1$