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1 $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

SOLN

$m^2 - m - 2 = 8$

$(m^2 - 2m) + 1(m-2)$

$m(m-2) + 1(m-2)$

$(m+1)(m-2)$

$m_1 = -1 \quad m_2 = 2$

CF : $y = Ae^{-x} + Be^{2x}$

PI : $y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$0 - 0 - 2(C) = 8$

$-2C = 8$

$C = -4$

GS : $y = Ae^{-x} + Be^{2x} - 4$

2 $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

SOLN

$m^2 - 4 = 0$

$m = \pm\sqrt{4}$

$m = \pm 2$

CF : $y = A\cosh 2x + B\sinh 2x$

PI : $y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

~~$9Ce^{3x} + 3Ce^{3x} - 4(Ce^{3x}) =$~~

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$c = 2$

PI : $y = 2e^{3x}$

GS : $C\cosh 2x + b\sinh 2x + 2e^{3x}$

3 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

SOLN

$m^2 + 2m + 1 = 0$

$(m^2 + m) + (m + 1) = 0$

$m(m+1) + 1(m+1)$

$(m+1)(m+1)$

$m = m_1 = m_2 = -1$

CF : $y = e^{-x}(A + Bx)$

PI : $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x}(4 - 4 + 1) = e^{-2x}$

$\frac{1Ce^{-2x}}{e^{-2x}} = \frac{e^{-2x}}{e^{-2x}}$

$C = 1$

PI : $y = e^{-2x}$

GS : $y = e^{-x}(A + Bx) + e^{-2x}$

4 $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

SOLN

$m^2 + 25 = 0$

$m = \pm\sqrt{25}$

$m = \pm 5$

CF : $y = A\cos 5x + B\sin 5x$

PI : $y = Cx^2 + bx + 5$

$\frac{dy}{dx} = 2Cx + b$

$\frac{d^2y}{dx^2} = 2C$

$2C + 25(Cx^2 + bx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25bx + 25E = 5x^2 + x$

$x^2[25C] + x[25b] + 2C + 25E = 5x^2 + x$

$25C = 5$ ——— (i)

$25b = 1$ ——— (ii)

$2C + 25E = 0$ ——— (iii)

From eqn (i)

$C = \frac{5}{25} = \frac{1}{5}$

From eqn (ii)

$b = \frac{1}{25}$

Substituting C in eqn (iii)

$2\left(\frac{1}{5}\right) + 25E = 0$

$\frac{2}{5} = -25E$

$E = \frac{2}{5} \times \frac{-1}{25}$

$E = \frac{-2}{125}$

PI : $y = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$= \frac{1}{125}(25x^2 + 5x - 2)$

GS : $A\cos 5x + B\sin 5x + \frac{1}{125}(25x^2 + 5x - 2)$

5 $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

Sol:

$m^2 - 2m + 1 = 0$

$(m^2 - m) - (m - 1) = 0$

$m(m-1) - 1(m-1) = 0$

$m = m_1 = m_2 = 1$

CF : $y = e^x(A + Bx)$

PI : $y = C\cos x + b\sin x$

$\frac{dy}{dx} = -C\sin x + b\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - b\sin x$

$-C\cos x - b\sin x - 2[-C\sin x + b\cos x] + C\cos x + b\sin x = 4\sin x$

$-C\cos x - b\sin x + 2C\sin x - 2b\cos x + C\cos x + b\sin x = 4\sin x$

$\cos x[-C - 2b + C] + \sin x[-b + 2C + b] = 4\sin x$

$-2b = 0$ ——— (i)

$2C = 4$ ——— (ii)

From eqn (i)

$b = 0$

From eqn (ii)

$c = \frac{4}{2}$

$c = 2$

PI : $y = 2\cos x$

GS : $y = e^{x}(A + Bx) + 2\cos x$

6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$, given that at $x = 0$, $y = 1$, $\frac{dy}{dx} = -2$

SOLN

$m^2 + 4m + 5 = 0$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm 2i}{2}$

$m = -2$

CF : $y = e^{-2x}[C\cos x + b\sin x]$

PI : $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$e^{-2x}[4C - 8C + 5C] = 2e^{-2x}$

$e^{-2x}[C] = 2e^{-2x}$

$C = 2$

PI : $y = 2e^{-2x}$

GS : $e^{-2x}[C\cos x + b\sin x] + 2e^{-2x}$

$1 = e^{-2(0)}[C\cos(0) + b\sin(0)] + 2e^{-2(0)}$

$1 = C + 2$

$1 - 2 = C$

$C = -1$

$-2 = e^{-2(0)}[-C\sin(0) + b\cos(0)] + [C\cos(0) + b\sin(0) - (2e^{-2(0)} + 2e^{-2(0)})]$

$-2 = b - 2C + 2$

$-4 = b - 2c$

$-4 = b - 2(-1)$

$-4 = b + 2$

$b = -6$

PS: $y = e^{-2x}[-c_0 x - 6\cos x] + 2e^{-2x}$

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 5$

SOLN

$\frac{1}{3}\left(3\frac{d^2y}{dx^2}\right) - \frac{1}{3}\left(2\frac{dy}{dx}\right) - \frac{1}{3}y = 0$

$\frac{d^2y}{dx^2} - \frac{2}{3}\frac{dy}{dx} - \frac{1}{3}y = 0$

$m^2 - \frac{2}{3}m - \frac{1}{3} = 0$

$m^2 - m + \frac{1}{3}m - \frac{1}{3} = 0$

$m(m-1) + \frac{1}{3}(m-1) = 0$

$m - 1 = 0 \qquad m + \frac{1}{3} = 0$

$m_1 = 1 \qquad m_2 = -\frac{1}{3}$

CF: $y = Ae^x + Be^{-\frac{1}{3}x}$

PI: $y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$0 - 2C - Cx - D = 2x - 5$

$-Cx - 2C - D = 2x - 5$

$-Cx = 2x$

$C = -2$

$-2C - D = -5$

$-2(-2) - D = -5$

$4 - D = -5$

$D = 9$

PI: $y = -2x + 9$

GS: $y = Ae^x + Be^{-\frac{1}{3}x} - 2x + 9$

8 $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Soln

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$m(m-4) - 2(m-4) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2 \quad m_2 = 4$

CF: $y = Ae^{2x} + Be^{4x}$

PI: $y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Ce^{4x} = 8e^{4x}$

$Ce^{4x}[16x + 4 + 4 - 24x - 6 + 8x] = 8e^{4x}$

$Ce^{4x}[+2] = 8e^{4x}$

$+2C = 8$

$C = +\frac{8}{2}$

$C = +4$

PI: $y = 4xe^{4x}$

GS: $y = Ae^{2x} + Be^{4x} + 4xe^{4x}$