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CIVIL ENGINEERING

ENG 381

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y - 8 = 0$

$m^2 - m - 2 = 0$

$m^2 + m - 2m - 2 = 0$

$(m^2 + m) + (-2m - 2) = 0$

$m(m+1) - 2(m+1) = 0$

$(m-2)(m+1) = 0$

$\therefore m_1 = 2, \quad m_2 = -1$

$y = Ae^{2x} + Be^{-x} = CF$

PI $\Rightarrow$ $y = C$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$0 - 0 - 2c = 8$

$-2c = 8$

$c = -4$

PI $\Rightarrow y = -4$

GS $= y = Ae^{2x} + Be^{-x} - 4$

2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$m^2 - 4 = 0$

$(m^2 + 2)(m - 2) = 0$

$m_1 = -2, \; m_2 = 2$

$m_1 = 2, \; m_2 = -2$

$y = Ae^{2x} + Be^{-2x}$

PI $\Rightarrow y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

$5C = 10$

$\therefore C = 2$

PI $\therefore y = 2e^{3x}$

GS $= y = Ae^{2x} + Be^{-2x} + 2e^{3x}$

(3) $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$(m^2 + m) + (m + 1) = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$(m+1) = 0$ twice

$m = -1$ twice

CF $= y = e^{-x}(A + Bx)$

PI $y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$$\frac{d^2y}{dx^2} = 4Ce^{-2x}$$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$$

$$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$$

$$C = 1$$

$$\therefore PI = y = e^{-2x}$$

$$GS = y = e^{-x}(A + Bx) + e^{-2x}$$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$$m^2 + 25 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0 \pm \sqrt{0^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{0 \pm \sqrt{-100}}{2}$$

$$= \frac{\pm j\sqrt{100}}{2}$$

$= \pm j10$

2

$= \pm j5$

$\therefore \alpha = 0, \beta = 5$

CF is $y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$\therefore y = e^{0}(A\cos 5x + B\sin 5x)$

$y = (A\cos 5x + B\sin 5x)$

PI $= y = Cx^2 + Dx + E$

$\frac{dy}{dx} = 2Cx + D$

$\frac{d^2y}{dx^2} = 2C$

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x + x$

$2C + 25E = 0 \quad \cdots (1)$

$25Cx^2 = 5x^2 \quad \cdots (2)$

$25Dx = x \quad \cdots (3)$

from eqn (3)

$25Dx = x, \quad D = 1/25 =$

from eqn (2)

$25Cx^2 = 5x^2$

$C = 5/25 = 1/5$

from eqn (3)

$2C + 25E = 0$

$2(1/5) + 25E = 0$

$2/5 + 25E = 0$

$\frac{2 + 125E}{5} = 0$

$2 + 125E = 0$

$E = \frac{-2}{125} = -0.016$

$C = 1/5, \; D = , \; E = -2/125$

$y = PI = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$\therefore GS = y = A\cos 5x + B\sin 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

6 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-x}$, given that at $x=0$, $y=1$ & $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2}$

$m = \frac{-4 \pm j\sqrt{4}}{2}$

$m = \frac{-4 \pm j2}{2}$

$= -2 \pm j$

$\alpha = -2, \beta = 1$

CF: $y = Ae^{\alpha x}(A\cos\beta x + B\sin\beta x)$

$y = e^{-2x}(A\cos x + B\sin x)$

$PI = y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{dy}{dx^2} = 4Ce^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5Ce^{-2x} = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$\therefore PI = y = 2e^{-2x}$

$GS = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

at $x = 0$, $y = 1$ & $\frac{dy}{dx} = -2$

(9)

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y = e^{-2(0)}(A\cos 0 + B\sin x) + 2e^{-2(0)}$

$1 = 1(A + 0) + 2$

$1 = A + 2$

$A = 1 - 2 = -1$

$\frac{dy}{dx} = [-2e^{-2x}(A\cos x + B\sin x) + e^{-2x}(-A\sin x + B\cos x)]$

$-4e^{-2x}$

1

at

$-2 = [-2e^{-2(0)}(A\cos 0 + B\sin 0) + e^{-2(0)}(-A\sin 0 + B\cos 0]$

$-4e^{-2(0)}$

$-2 = [-2(A+0) + 1(0+B) - 4$

$A = 1/2 - 1$

$-2 = [(-2A + B) - 4]$

$-2 = [-2(\frac{1}{2}) + B - 4]$

$-2 = -1 + B - 4$

$-2 = B - 5 \quad -2 = -2 + B$

$B = -2 + 5 \quad B = -2 + 2$

$B = 3 \quad = 0$

$A = -1, B = 0$

$\therefore GS = y = e^{-2x}(-1\cos x + 0\sin x) + 2e^{-2x}$

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

(5) $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$(m^2 - m)(m + 1) = 0$

$m(m+1) + 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = 1$ twice

$y = e^x(A + Bx) = CF$

$PI = y = C\cos x + D\sin x$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 4\sin x$

$(-C\cos x - D\sin x) - 2(-C\sin x + D\cos x) + (C\cos x + D\sin x) = 4\sin x$

$= -C\cos x - D\sin x + 2C\sin x - 2D\cos x + C\cos x + D\sin x = 4\sin x$

$= -C\cos x + C\cos x - D\sin x + D\sin x + 2C\sin x - 2D\cos x = 4\sin x$

$2C\sin x - 2D\cos x = 4\sin x$

$2C = 4$

$C = 4/2$

$C = 2$

$-2D = 0$

$D = 0/-2$

$= 0$

$\therefore C = 2, \quad D = 0$

$PI = y = 2\cos x + 0\sin x$

$y = 2\cos x$

$GS = e^x(A + Bx) + 2\cos x$

7 $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$(3m^2 + 3m) + (m - 1) = 0$

$3m(m-1) + 1(m+1) = 0$

$(3m+1)(m-1) = 0$

$m = -1/3, \quad m = 1$

$m_1 = 1, \quad m_2 = -1/3$

$CF = y = Ae^{x} + Be^{\frac{1}{3}x}$

$PI = y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3(0) - 2(C) - (Cx + D) = 2x - 3$

$0 - 2C - Cx - D = 2x - 3$

$-2C - D = -3 \quad \cdots (1)$

$-Cx = 2x \quad \cdots (2)$

from eqn (2)

$-Cx = 2x$

$C = -2$

from eqn (1)

$-2C - D = -3$

$-2(-2) - D = -3$

$-4 - D = -3$

$-D = -3 + 4$

$-D = -7$

$C = -2, \; D = 7$

$y = Cx + D$

$PI = y = -2x + 7$

$GS = y = Ae^{x} + Be^{-1/3x} - 2x + 7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$(m^2 - 4m) - (2m + 8) = 0$

$m(m-4) - 2(m-4) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2, m_2 = 4$

$y = Ae^{2x} + Be^{4x} = CF$

$PI = y = Ge^{4x}$

$\frac{dy}{dx} = 4Ge^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Ge^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$$

$$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8Cxe^{4x} = 8e^{4x}$$

$$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$$

$$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$$

$$8Ce^{4x} - 6Ce^{4x} = 8e^{4x}$$

$$2Ce^{4x} = 8e^{4x}$$

$$2C = 8$$

$$C = 8/2$$

$$C = 4$$

$$PI = y = 4xe^{4x}$$

$$GS = CF + PI$$

$$= Ae^{2x} + Be^{4x} + 4xe^{4x}$$