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ENG 281
Mech ENG
16/ENG06/043

Assignment

1a/ Evaluate $\lim_{x \to \pi/2} \left\{ \frac{x^2 - \pi/4 \sin(\cos x)}{x - \pi/2} \right\}$

$$= \lim_{x \to \pi/2} \left\{ \frac{(2x - 0)(\sin)(-\sin x) + \cos x(\cos x)}{1 - 0} \right\}$$

$$= \lim_{x \to \pi/2} \left\{ \frac{2x(-\sin^2 x + \cos^2 x)}{1} \right\}$$

$$= 2 \cdot \pi/2 \quad (-\sin^2(\pi/2) + \cos^2(\pi/2)]$$

$$= \pi(-1+0)$$

$$= \pi(-1)$$

$$= -\pi$$

$$\therefore \lim_{x \to \pi/2} \left\{ \frac{(x^2 - \pi/4)(\sin(\cos x)}{x - \pi/2} \right\}$$

$$= -\pi$$

b) $\lim_{x \to \pi/2} \ln \left\{ \exp \frac{(3x^n + nx - 1)}{x + 1} \right\}$

$$\lim_{x \to \pi/2} \left\{ \frac{(3x-1)(x+1)}{(x+1)} \right\}$$

$$= \lim_{x \to \pi/2} (3x - 1)$$

$$= 3(\pi/2) - 1$$

$$= 3\pi/2 - 1$$

$$= \frac{3\pi - 2}{2}$$

c.) $\lim_{x \to 2+\sqrt{3}} \cos \left\{ \frac{\sin^{-1}(x-2)}{(x-\sqrt{3})} \right\}$

$$\lim_{x \to 2+\sqrt{3}} \frac{\cos \sin^{-1}(x-2)}{\cos \sin^{-1}(x-\sqrt{3})}$$

$$= \cos \left\{ \frac{\sin^{-1}(2+\sqrt{3}-2)}{2+\sqrt{3}-\sqrt{3})} \right\}$$

$$= \cos\left\{\sin^{-1}\frac{\sqrt{3}}{2}\right\}$$

$$= \cos 60 = \frac{1}{2}$$

1d.) $\lim_{x \to 4} \left\{ \frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right\}$

$= \frac{4^2 - 8(4) + 16}{4^2 - 5(4) + 4} = \frac{0}{0}$

$\lim_{x \to 4} \frac{2x - 8}{2x - 5}$

$= \frac{2(4) - 8}{2(4) - 5} = \frac{0}{3} = 0$

$\therefore \lim_{x \to 4} \left\{ \frac{x^2 - 8x + 16}{x^2 - 5x + 4} \right\} = 0$

2 a.) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \dots$

$$U_n = \frac{2}{(n+1)\times(n+2)}$$

$$U_{n+1} \quad \frac{2}{(n+2)\times(n+3)}$$

$$\lim_{n\to 0} \frac{U_{n+1}}{n} = \frac{2}{(n+2)(n+3)} \times \frac{(n+3)(n+2)}{2}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{n^2+3n+2}{n^2+5n+6}$$

Divide through by highest O+n as $n\to\infty$, $5n/n^2 \to 0$,

$2/n^2 \to 0 \quad 5/n^2 \to 0 \quad 0/n^2 \to 0$

$$\lim_{n\to\infty} = \frac{1 + 3n/n^2 + 2/n^2}{n^2/n^2 + 5/n^2 + 6/n^2}$$

$$\frac{1+0+0}{1+0+0} = 1$$, from the rule It may converge or diverge.

From test 1 $\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{2}{n^2+3n+2)} = \frac{2/n^2}{n^2/n^2 + 3n/n^2 + 2/n^2}$

$0/1 = 0$ The series is convergent

∴ ~~The series is (~~

2b.) $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$

$U_n = \frac{2}{n^2}$, $U_{n+1} \frac{2}{(n+1)^2}$

$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{n^2+2n+1}{n^2}$

$\frac{U_{n+1}}{U_n} = \frac{n^2/n^2 + 2n/n^2 + 1/n^2}{n^2/n^2}$

$= \frac{1+0+0}{1} = 1$

From the rule it may converge or diverge

From Test 1 $\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{2}{n^2}$

$\frac{2/n^2}{n^2/n^2} = \frac{0}{1} = 0$

∴ The series is convergent

2c.) $u_n = \frac{1+2n^2}{1+n^2}$

$$\lim_{n\to\infty} u_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$$

$$= \frac{0+2}{0+1} = \frac{2}{1}$$

$= 2$ ∴ The series is divergent.

3.) $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$

$u_n = \frac{x^n}{(2n+1)^3}$, $u_{n+1} = \frac{x^{n+1}}{2(n+1)+1)^3} = \frac{x^{n+1}}{(2n+3)^3}$

$$\lim_{n\to\infty} = \frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= \frac{(x)(2n+1)^3}{(2n+3)^3}$$

$$= \frac{(8n^3+12n^2+6n+1)x}{8n^3+18n^2+54n+27}$$

$$= \frac{8xn^3+12xn^2+6xn+x}{8n^3+18n^2+54n+27}$$

divide by highest power of n.

$$= \frac{\frac{8xn^3}{n^3} + \frac{12xn^2}{n^3} + \frac{6xn}{n^3} + \frac{x}{n^3}}{8n^3/n^3 + 18n^2/n^3 + 54n/n^3 + 27/n^3}$$

$$= \frac{8x}{8}$$

$$= \frac{8x}{8} < 1$$

$8x < 8$

$x < 1$

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4.) $\lim_{x \to 0} \left\{ \frac{\sin x - \cos x}{x^3} \right\}$

$\frac{\sin 0 - \cos 0}{0^3} = \frac{0-1}{0}$ undefine

$\lim_{x \to 0} \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1+0}{0}$ undefine

$\lim_{x \to 0} \frac{-\sin x + \cos x}{6x} = \frac{-\sin 0 + \cos 0}{6(0)} = \frac{-0+1}{0}$

undefine

$\lim_{x \to 0} = \frac{-\cos x - \sin x}{6} = \frac{-\cos 0 - \sin 0}{6} = \frac{-1-0}{6}$

$= \frac{-1}{6}$

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