NAME: ONYEKA HENRY CHINEME 16/ENG04/045
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(a) Evaluate $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right]$

$\Rightarrow$ Product rule $V\frac{du}{dx} + U\frac{dv}{dx}$; $V = x^2 - \pi/4$ $U = \sin(\cos x)$

$\frac{du}{dx} \Rightarrow P = \cos x$ $\qquad U = \sin P$

$\frac{dP}{dx} = -\sin x$ $\qquad \frac{du}{dP} = \cos P$

$\frac{du}{dx} = -\sin x \cdot \cos x(\cos x)$

$V = x^2 - \pi/4$; $\frac{dv}{dx} = 2x$

$\Rightarrow \frac{(x^2 - \pi/4)(-\sin x \cos(\cos x)) + \sin(\cos x)(2x)}{1}$

$\lim_{x \to \pi/2} \Rightarrow ((\pi/2)^2 - \pi/4)(-\sin(\pi/2) \cdot \cos(\cos \pi/2)) + \sin(\cos \pi/2)(2 \times \pi/2)$

$= (\pi^2/4 - \pi/4)(-1 \cos(0))$

$= \frac{\pi^2}{4} - \frac{\pi}{4} \times -1$

$= \frac{-\pi^2}{4} + \frac{\pi}{4}$

$\Rightarrow \lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)(\sin(\cos x))}{x - \pi/2} \right] = \frac{-\pi^2}{4} + \pi/4$

b) $\lim_{x \to \pi/2} \ln\left[\exp \frac{(3x^2+2x-1)}{x+1}\right]$

$\Rightarrow$ factorize: $(3x^2+2x-1)$

$\Rightarrow (3x-1)(x+1)$

$\Rightarrow \ln\left[\exp \frac{(3x-1)(x+1)}{x+1}\right]$

$\ln\left[\exp(3x-1)\right]$

$\lim_{x \to \pi/2} \Rightarrow \ln\left[\exp\left(\frac{3\pi}{2}-1\right)\right]$

$\lim_{x \to \pi/2} \ln\left[\exp\frac{(3x^2+2x-1)}{x+1}\right] = \frac{3\pi}{2} - 1$

c) $\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right]$

$\Rightarrow \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$

$\cos\left[\sin^{-1}\left(\sqrt{3}/2\right)\right]$

$\cos\left[60^\circ\right]$

$= 1/2$

$\lim_{x \to 2+\sqrt{3}} \cos\left[\frac{\sin^{-1}(x-2)}{(x-\sqrt{3})}\right] = 1/2$

d) $\lim_{x \to 4} \left[\dfrac{x^2 - 8x + 16}{x^2 - 5x + 4} \right]$

$\Rightarrow$ factorize:

$\lim_{x \to 4} \left[\dfrac{(x-4)\cancel{(x-4)}}{(x-1)\cancel{(x-4)}} \right]$

$\lim_{x \to 4} \left[\dfrac{x-4}{x-1} \right]$

$\Rightarrow \dfrac{4-4}{4-1}$

$\lim_{x \to 4} \left[\dfrac{x^2 - 8x + 16}{x^2 - 5x + 4} \right] = 0/3 = 0$

2) (a) $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6}$

Compare to Standard Converging Series

$\Rightarrow \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \frac{1}{5^p} + \frac{1}{6^p} + \frac{1}{7^p} + \cdots$

When $P = 2$

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots$

$\frac{2}{1\times3} < \frac{1}{1^2}$, $\frac{1}{2\times3} < \frac{1}{2^2}$, $\frac{2}{4\times5} < \frac{1}{3^2}$, $\frac{2}{5\times6} < \frac{1}{4^2}$

Since each term is less than its corresponding term in the series;

$\therefore$ The series CONVERGES

b) $2/1^2 + 2/2^2 + 2/3^2 + 2/4^2 + \dots$

$\Rightarrow U_n = 2/n^2 \quad U_{n+1} = 2/(n+1)^2 \quad U_{n+1} = 2/n^2+2n+1$

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{n^2+2n+1}{n^2}$$

$$= \frac{n^2/n^2 + 2n/n^2 + 1/n^2}{n^2/n^2}$$

$$= \frac{1+(2\times 0)/0 + 1/0}{1}$$

$$= \frac{1+0+0}{1} = 1$$

From the rule the series may convergent or divergent

from test 1 $\lim_{n \to \infty} U_n = \lim_{n \to \infty} 2/n^2$

$= \cancel{4/n^2}$

$= \frac{2/n^2}{n^2/n^2} = 0/1 = 0$

$\therefore$ This Series CONVERGES

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$$U_n = \frac{1+2n^2}{1+n^2}$$

$$\lim_{n \to \infty} U_n = \frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}$$

$$\frac{0+2}{0+1} = 2/1$$

$$= 2$$

$\therefore$ This series DIVERGES

3) $\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$

$U_n = x^n/(2n+1)^3$, $U_{n+1} = x^{n+1}/(2(n+1)+1)^3 = x^{n+1}/(2n+2+1)^3$

$$= \frac{x^{n+1}}{(2n+3)^3}$$

$$\frac{U_{n+1}}{U_n} = \frac{x^n * x}{(2n+3)^3} \times \frac{(2n+1)^3}{x^n} = \frac{x(2n+1)^3}{(2n+3)^3}$$

$$\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right| = \lim_{n \to \infty} x \frac{(8n^3 + 12n^2 + 6n + 1)}{8n^3 + 36n^2 + 54n + 27}$$

$$\lim_{x \to \infty} x = \frac{(8n^3/n^3 + 12n^2/n^3 + 6n/n^3 + 1/n^2)}{8n^3/n^3 + 18n^2/n^3 + 54n/n^3 + 27/n^3}$$

$$= x \times 8/8 = x$$

$\therefore x \leq 1$

-) $\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$

$\Rightarrow \frac{\sin 0 - \cos 0}{0^3} = \frac{0 - 1}{0}$ undefined

$\lim_{x \to 0} \frac{\cos 0 + \sin 0}{3(0)^2} = \frac{1+0}{0}$ undefined

$\lim_{x \to 0} \frac{-\cos x - \sin x}{6} = \frac{\cos 0 - \sin 0}{6}$

$= \frac{-1 - 0}{6} = -1/6$