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ELECT/ELECT
ENG 281

Assignment

The parametric equation of a curve are as given as equation (1) and (2)

$x = \cos t + t \sin t$ (1)

$y = \sin t - t\cos t$ (2)

In terms of t, determine
i) an expression for the radius of curvature (R)

$$\Rightarrow \quad R = \frac{\left[1 + \left[\frac{dy}{dx}\right]^2\right]^{3/2}}{d^2y/dx^2}$$

$x = \cos t + t\sin t$

$dx/dt = -\sin t + t\cos t + \sin t$

$dx/dt = t\cos t$

$y = \sin t - t\cos t$

$dy/dt = \cos t - (-t\sin t + \cos t)$

$dy/dt = \cos t - (-t\sin t + \cos t)$

$= \cos t + t\sin t - \cos t$

$= t\sin t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= t\sin t \times \frac{1}{t\cos t} = \frac{t\sin t}{t\cos t} = \frac{\sin t}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \times \frac{dt}{dx}$$

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$$\frac{d^2y}{dx^2} = \frac{V\,{du}/{dx} - U\,{dv}/{dx}}{V^2}$$

where $U = \cos t$

$du = \sin t$

$U = \sin t$

$$\therefore \frac{d^2y}{dx^2} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \times \frac{dt}{dx}$$

recall; $\cos^2 t + \sin^2 t = 1$

$$\frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$$

Also recall that

$$R = \frac{[1 + [{dy}/{dx}]^2]^{3/2}}{{d^2y}/{dx^2}}$$

$$R = \frac{[1 + ({\sin t}/{\cos t})^2]^{3/2}}{1/\cos^3 t}$$

$$R = (1 + {\sin^2 t}/{\cos^2 t})^{3/2} \times {t\cos^3 t}/{1}$$

$$R = ((\cos^2 t + \sin^2 t)/\cos^2 t) \times t\cos^3 t$$

$$R = \frac{1}{(\cos^3 t)^{3/2}} \times t\cos^3 t = \frac{1}{(\sqrt{\cos^2 t})^3} \times t\cos^3 t$$

$$R = \frac{t\cos^3 t}{\cos^3 t} = t$$

$\therefore$ The radius of curvature is t

b) Expressions for the co-ordinates (h, k) of the centre of curvature

$\Rightarrow$ $h = x_1 - R\sin\theta$ ①

$k = y_1 + R\cos\theta$ ②

$R = t\ t = \theta$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t + t\cos t$

Substituting x_1 in eqn ① and y in eqn ②

$h = \cos t + t\sin t - t\sin t$

$h = \cos t$

$k = \sin t - t\cos t + t\cos t$

$k = \sin t$

The equations for the coordinates (h, k) of the centre of curvature is $(\cos t, \sin t)$